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MATHEMATICS INDIVIDUAL LEARNING EXPERIMENT.
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THIS EXPERIMENT TESTED AND EVALUATED THE CHANGE IN ACHIEVEMENT AND ATTITUDE TOWARD MATHEMATICS OF NINTH-GRADE, ELEMENTARY ALGEBRA PUPILS SUBJECTED TO TWO METHODS OF INSTRUCTION. TWO HYPOTHESES WERE TESTED--(1) CLASSES TAUGHT BY INDIVIDUALIZED AND TRADITIONAL METHODS DO NOT DIFFER IN NINTH-GRADE ALGEBRA ACHIEVEMENT AND (2) INDIVIDUALIZED AND TRADITIONAL METHODS OF TEACHING NINTH-GRADE ALGEBRA SHOW NO DIFFERENCE IN CHANGING ATTITUDES TOWARD MATHEMATICS. AN INSTITUTE WAS HELD FOR ORIENTATION OF SELECTED TEACHERS. THE RANDOMIZED BLOCK EXPERIMENT INVOLVED 36 NINTH-GRADE ELEMENTARY ALGEBRA CLASSES IN NINE JUNIOR HIGH SCHOOLS IN THE NEW YORK CITY AREA. BOTH STUDENT AND TEACHING METHOD WERE ASSIGNED AT RANDOM. THE ATTITUDE ANALYSIS REVEALED A SIGNIFICANT DROP IN MEAN SCORES OF BOTH GROUPS FROM THE BEGINNING TO THE END OF THE TERM. NO SIGNIFICANT DIFFERENCES WERE FOUND BETWEEN THE TWO GROUPS IN ACHIEVEMENT. (JK)

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MATHEMATICS INDIVIDUAL LEARNING EXPERIMENT

Gwladys Crosby
Herbert Fremont
Harold E. Mitzel

Title VII Project Number 391

National Defense Education Act of 1958

Grant Number 7-42-1050-069 & 7-30-06900

Principal Investigator: Gwladys Crosby

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ABSTRACT OF RESEARCH REPORT

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Purpose

The chief purpose of this investigation was to evaluate, experimentally, the growth in achievement and attitude toward mathematics of ninth grade, elementary algebra pupils subjected to two methods of instruction. A pilot study, based on the assumption that it is both possible and practical to individualize instruction in heterogeneous classes, resulted in the development of a technique which seemed to show promise. Fundamental to that technique was the concept that pupils must: participate in goal setting, plan their work within the framework of the syllabus and their interest and abilities, request tests when ready, and participate in evaluation of their work. In addition it had been found that perceptual devices consisting of concrete materials, usually referred to as teaching aids, were needed for adequate application of the experimental method. In order to compare the effectiveness of that technique, hereafter called the experimental method, with traditional methods characterized by teacher control of goal setting, work assignments, testing and evaluation, an experiment was performed to test the following hypotheses: -

1. A universe of classes taught by individualized and traditional methods do not differ in ninth grade algebra achievement.

2. A universe of classes taught ninth grade algebra by individualized and traditional methods do not differ with respect to changes in attitude toward mathematics.

Procedure

In order to prepare teachers to carry out the experiment an institute was held at Queens College, Flushing, N.Y., during the summer of 1960, at which time the teachers:-

(1.) were oriented to experimental procedures.
(2.) constructed perceptual devices.
(3.) practiced applying the experimental method.

In September the classroom phase began. The completely randomized block experiment involved 36 ninth grade, elementary algebra classes in nine junior high schools in the New York metropolitan area. Randomization for the validation of experimental error stemmed from two sources. First, the ninth grade pupils in any given school who enrolled for the first course in algebra were assigned by a random method to one of four classes. Second, the method to be used by the teachers was assigned to the particular class in random fashion. Because this experiment was designed to compare the changes in achievement and attitude on the part of pupils who were taught by two different methods, four variables were employed in covariance, and analysis of variance designs. The two dependent variables were pupil achievement in algebra as measured by a specially constructed 117-item examination, and the end-of-year score of each pupil on a 50 item Attitude Toward Mathematics Scale. The independent variable used as a covariant item in the analysis of the achievement data was a composite T-score based on Form A of four sub-tests of the Differential Aptitude Tests. Six of the nine original schools repeated the attitude inventory at the end of the school year. In order to meet the condition of equal cell frequencies, cases were withdrawn by a random procedure until each of the classes in the analysis contained exactly 18 pupils, each with beginning and end-of-term attitude scores.

RESULTS AND CONCLUSIONS

Failure of the F-ratio in the adjusted achievement analysis to meet the five per cen significance level lead to acceptance of the null hypothesis and the conclusion that the observed difference between adjusted means for methods were consistent with chance findings. The attitude analysis revealed a significant drop in mean scores of pupils taught both methods from the beginning to the end of the term.

It would be erroneous to conclude from the above results, that the two methods are equally effective in promoting achievement, or that both are ineffective in improving students' attitudes. Teachers, principals and administrators found it difficult to fulfi adequately the requirements of the experimental design. As a result it is possible that many small deviations from design occurred; failure to drop schools in which the describe differences between experimental and control groups were not being maintained may have influenced the results. For this reason further study is clearly indicated. To provide adequate controls it is important that teachers be involved as researchers and that they be competent in both methods of instruction. In addition, pre and post experiment attitudes of teachers toward both children and methods must be measured, as must the degree of individualization present in the classroom. A pressing need is for the development of techniques which will permit analysis of individual cases, since computation of a mean result in absolute anonymity of all the contributing elements. There appears to be a a paradox the use of a measure of central tendency to test hypotheses founded on the assumption tha differences between individuals should be magnified rather than minimized by the educatio al process. In addition, attempts should be made to develop meaningful operational defin tions of variables that will reflect all of the educational objectives inherent in instruction.

Despite the inadequacies of the present experiment, it is possible that significant differences do exist between the two methods and thus the present investigators must conclude that this study should be replicated under conditions which would permit fidelity to the original experimental design.

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Mr. Roland Genise, Supervisor, Brentwood Mathematics Center

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Mrs. Thelma Hickerson, Asst. Principal
Mr. Joseph Biegen, Principal

Carr Junior High - Mr. Morris Bunsis, Teacher,
Mr. Jack Graulick, Asst. Principal
Dr. Hyman Birnbaum, Principal

Port Jefferson Junior High
Mr. Richard Shaughnessy, Teacher,
Mrs. Frances Terry, Math Dept. Chairman
Mr. Edward Wittemeier, Principal

Pulitzer Junior High - Mrs. Helen Keefe, Teacher,
Dr. Bernard Rush, Asst. Principal
Dr. Moses Nagelberg, Principal

Shimer Junior High - Mr. Vincent Smyth, Teacher
Mr. William Goldschlager, Asst. Principal
Mrs. Edna V. Crowley, Principal

Stanforth Junior High - Mr. Alexander Imre, Teacher,
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Mr. Alton D. Lowe, Building Principal

Syosset High School - Mr. Arthur Beckman, Teacher,
Mr. Edward Augustine, Director of Guidance,
Dr. William French, Principal

Willoughby Junior High- Mr. Paul Medici, Teacher,
Mr. Edward Malament, Asst. Principal
Mrs.Leonore G. Berman, Principal

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CHAPTER I

DEVELOPMENT AND EVALUATION OF AN INDIVIDUALIZED APPROACH IN THE TEACHING OF NINTH GRADE ALGEBRA

There is no need to document the hysteria generated by Russian success in "space projects" nor to emphasize their immediate impact on education. Public guilt seemed to demand a scape-goat and a wave of criticism engulfed mathematics education especially; teachers, in particular, were declared inadequate, although students and curricula were also attacked. In response, educators increased their attempts to arouse interest and secure proficiency in mathematics. These attempts had led to ability grouping, track systems and "acceleration" -- administrative devices based on the twin-assumption that individualization of instruction within heterogeneous groups is impossible and that the learning process operates most effectively when individual differences are eliminated, or at least minimized, by reduced heterogeneity of classes. Effort, therefore, has been concentrated on increasing the homogeneity of classes, with little attention directed to consideration of the efficacy of the lecture-recitation method of instruction.

Significance

Knowledge of individual differences seems to indicate that the solution to the dual problem of interest and mastery must be found in individualization of instruction. The famed Dalton Plan[1] was a result of such thinking. The idea of individualization is, however, frequently accepted in theory while rejected in practice. In other words, it sounds good but is impractical; by "impractical" is meant too difficult or too time-consuming for the teacher. Obviously the fact that a method of instruction is difficult is not, in itself, reason for rejection. It would seem essential, before any method is dismissed with the label impractical, to examine the assumptions upon which it is based, to devise techniques for enabling teachers to apply it, and to design experiments to evaluate its effectiveness. To that end, the experiment reported here was planned. Two methods of instruction in ninth grade algebra are compared. The result of such research, while not necessarily conclusive in itself, should lead to replications and to the development of hypotheses which might be tested more widely; thus it could contribute significantly to education as a whole.

For almost a century there has been sporadic effort to develop techniques for individualization of instruction in one academic area or another. (See Ch. III) Most of these efforts were deemed effective by their originators but none have become accepted practice. In mathematics education, the development of methods of individualization have, since shortly after World War II, been neglected in favor of concentration on the curriculum and preparation of teachers in "Modern Mathematics."[2] There have been so many developments in mathematics unreflected in secondary curricula that mathematicians and mathematics educators have recently been chiefly concerned with curriculum reform. A number of groups have developed secondary curricula which are being tried out by students and teachers in various parts of the country; these groups are concerned with "modern" as opposed to "traditional" mathematics and in algebra the chief effort is to present a new approach to the subject matter using the concept of "set" as a unifying agent while emphasizing the structure of the real number system.

The need for these changes in curricula and the widespread conviction that mathematics was badly taught were bound to have some effect. The National Science Foundation has offered summer institutes at colleges and universities throughout the United States where teachers take courses designed to increase their knowledge of mathematics and thus improve their teaching ability. Apparently no experimental work is being done to measure teacher-effectiveness "before and after" institute training although a few "studies" have been reported.[3]

To state that there is little experimentation does not negate the fact that great effort is being made to improve mathematics teaching. On a priori grounds, curriculum revision and improved academic preparation of teachers are considered the keys to such improvement.

Need

On the whole it might be said that personal opinion, and even wishful thinking, have too often been the basis for recommended change in mathematics education. With the publication of B. F. Skinner's article on teaching machines, teachers could no longer avoid facing his assertion that adequate "programs" would succeed, where teachers had failed, in individualizing instruction.[4] On the other hand, teachers who had developed techniques of individualization characterized primarily by pupil freedom in the choice of reading material based their work on theoretical assumptions quite different from these characteristics of reinforcement theory.[5] From reinforcement theory in psychology then, came a need for the present study; another came from the reported successes of individualized approaches to reading instruction in the elementary grades. In summary it might be said that the present study is needed precisely because so many sweeping changes in curricula and teacher preparation are being made on the basis of rational assumptions, when they should be, in this day and age, subject to prior experimentation.

CHAPTER II - THE PROBLEM

The major problem this research was designed to answer was: How can instruction be individualized in ninth year algebra so that each student learns as much as he can, as fast as he can?

In order to answer this question it was necessary first, to define individualized and traditional instruction in order to clarify their differences. Second, it was essential to develop perceptual materials for classes taught by the individualized method. An attempt had been made in a pilot project to develop techniques of individualizing instruction and at that time it was concluded that such materials could help some student discover mathematical principles which they could not discover through use of the text book alone.

Since the question of attitude is intrinsic to any methods comparison, it was necessary to construct an attitude inventory in order to compare changes in attitude toward mathematics, and it was necessary, of course, to plan to compare the academic achievement of pupils taught by traditional and individualized methods. Underlying all of this was the need to retrain teachers so that they could carry out a rigorous plan for contrasting two methods of instruction.

Definitions

Individualized instruction is defined as that in which each pupil participates in setting his own goals, works at his own rate (either alone or as a member of a small group) utilizing sets of perceptual materials designed to facilitate discovery of algebraic principles, and participates in evaluating his own progress.

Traditional instruction is defined as all methods in which pupils are taught as a class. It includes homogeneous or heterogeneous grouping and does not preclude the use of audio-visual aids, committee work, or any other techniques traditionally used by teachers to help students learn.

By "small groups" is meant a sub-group of a class, usually three to five pupils.

Attitude toward mathematics in this study is defined operationally as the score obtained on a 50-item Attitude toward Mathematics Inventory. (See Appendix B).

Perceptual materials are defined as physical devices capable of conveying meaning through one or more sense organs. Examples are manipulative devices, tapes, puzzles, games, and physical experiments performed by pupils.

Assumptions

The thesis that the solution to the dual problem of interest and mastery must be found in individualized instruction within the heterogeneous classes has usually been dismissed as unrealistic, not on psychological grounds but because of the difficulties of implementation. There are, however, certain assumptions related to learning, which psychologists and teachers alike have long accepted, and which lead to the inevitable conclusion that the uniqueness of each child in a class will result in differences in both quantity and quality of learning regardless of teachers' attempts to teach a class as a whole. It was the general, if somewhat vague, recognition of the importance of individual differences in relation to learning which undoubtedly led to previous attempts to individualize instruction. The specific recognition of the role of reinforcement theory in psychology of learning lead to the attempts of Skinner and his adherents to program various curricula and of teachers and administrators to look to teaching machines for the solution to the problems of individualization. The following assumptions lead to the conclusion that more than the rate of learning must be taken into account in formulating a plan of individualized instruction:

1. Motivation to learn originates in basic needs which each individual seeks to satisfy in ways unique to himself.
2. Opportunities for perceptual experiences influence the apparent intelligence of an individual.
3. Materials which stimulate sensory organs are aids to learning. Individuals differ in their ability to perceive by various senses; i.e., some people tend to learn primarily by visual experiences, others by auditory, others by kinesthetic.
4. Insight and understanding are dependent upon a learner's opportunity to discover relationships, to discriminate cues and to re-structure his environment in terms of his unique perceptions.
5. Both maturation and previous experience are requisites of cognition.
6. Heredity and experience are unique to each individual, therefore learning is a highly individual process.
7. A democratic classroom atmosphere is more appropriate than either autocratic or laissez-faire for the education of students who live in a democracy.
8. Responsibility for learning eventually reposes in the learner. Freedom to learn from one's mistakes (without fear of punishment) leads to acceptance of such responsibility.

Description of Approach

On the basis of the above assumptions, individualization of instruction as defined in this experiment required that students participate in setting their goals. Furthermore, it was important that they plan to reach those goals within certain time-periods; this required that they take responsibility for scheduling and completing their own learning. Finally, for full understanding of achievement and for a genuine feeling of accomplishment, they must participate in evaluation of their own learning. A description of the individualization process in operation would begin with the teacher and class face to face on the first day of school. The teacher would immediately involve the students in planning unless, for some reason, he felt that they were not ready to undertake such a step because of their previous experience with authority. In any case, he would begin by talking with the pupils about the goals of the course. At the end of a number of sessions, approximately two weeks, every child would have his own plan of work for the year, with appropriate segments as immediate goals; the teacher's active participation would help students to set realistic levels of aspiration although the plan of work, being stated in units of time, would be flexible.

Having set their goals and planned their work, students would be free to work alone or with other students in attempting to reach those goals. Most often students would form small groups based on friendship patterns; sometimes one of these groups will be composed of five students who are working on the same "unit" while another group might be composed of five students each working on a different unit. It is important for the teacher to help keep these groupings flexible since one of the goals of a democratic classroom is that pupils help each other over brief hurdles. In the film accompanying this report there is a demonstration of the casual way in which a teacher requests a child to leave his group to help a classmate having a temporary difficulty. Shortly after the groups begin to function as scholars, workers, and helpers-of-each-other, the teacher will find that a child or a group does not understand an explanation on the textbook. It is at this point that the teacher either actively participates in stimulating or suggests another perceptual device (the textbook is, of course, the first and primary perceptual device) which will clarify some concept or principle. For example, if a student is having difficulty with ratio problems an appropriate device (see Appendix D) might be suggested by the teacher. If the child still does not understand, an explanation on tape may be suggested. In every instance, however, the teacher avoids giving a pupil the kind of information which will destroy the discovery aspect of the students' activities.

As students feel that they have mastered a unit of work they are free to request teacher-made tests. A unique feature of the individualized instruction based on the psychological principles discussed on page 4 is that the student is permitted to learn from his failures rather than being punished for them. Consequently, if a student misjudges his readiness and fails a test he merely throws it away and begins to prepare for another. Students keep their own records of test scores during the marking period; when it comes time for report cards there is no anxious waiting to see what the teacher will "give" them. The teacher's role throughout is that of guide, helper, and leader.

Figure I is designed to help clarify the differences between the two methods being compared. An analysis is herein presented of these differences in terms of goals, tasks, evaluations and tests.

	Group Instruction	Individual Instruction
GOALS	Set by teacher in terms of syllabus and text.	Set by teacher and pupil within framework of syllabus, after group discussion and individual conferences.
TASKS	Classwork Assigned by teacher on basis of time, syllabus, and text.	Assigned by pupil in terms of goals, after group planning and individual planning.
	Homework Assigned by teacher on the basis of pupil needs.	Assigned by pupil on basis of felt need for practice, or confidence in ability to proceed alone.
EVALUATION	Done by teacher on basis of classwork, tests, and other teacher selected criteria.	Done by teacher and pupil based on progress toward goal determined by tests, and subjective discussion.
TESTS	Constructed and administered by teacher, scored on basis of teacher's need for evaluation.	Made by teacher, pupil requested when ready. Scored jointly.

Figure I

Hypotheses

Having developed the technique of individualization it was necessary to compare classes taught by that method with those taught by a traditional method. To make such a comparison valid, it was important to design an experiment in which as many variables as possible could be controlled. The next chapter will contain a review of the literature and in Chapter IV we shall return to the experimental design. Before closing this chapter, however, we turn to an explicit statement of those hypotheses this experiment was designed to test.

1. A universe of classes taught by individualized and traditional methods do not differ in ninth grade algebra achievement.

2. A universe of classes taught by individualized and traditional methods do not differ with respect to changes in attitude toward mathematics.

CHAPTER III - RELATED RESEARCH

Studies which report experimental work on individualization of instruction in mathematics through the use of perceptual materials for discovery have not been found in the literature; studies which are related to the present investigation by virtue of their similarity to at least one of its aspects are reported below.

The first published record of individualized instruction appeared in 1894.[1] In his article describing the "Pueblo Plan," W. P. Search, its originator, gave his impressions of the results of individualization; he had made no attempt to conduct what could be called an education experiment. The work is mentioned here primarily becuase of its historical significance and because it lends support to the statement that the problem of individualization has been with us for the greater part of a century.

In 1912, at the San Francisco State Normal School Experimental Elementary School, Burk & Ward developed the first definite techniques of individual instruction in America.[2] Burk's plan involved the elimination of class recitations, with each student working at his own rate in each subject by using self-instruction pamphlets created for this purpose. Careful individual records were kept for each student. A study of these records showed that 100 out of 130 pupils had saved time by working on the individual plan. Over 75 per cent of the students completed more than a grade's work in a year.

Burk's system,adopted by some schools, met much resistance in city school systems. Carleton Washburne, Superintendent of Schools in Winnetka, Illinois, who had been a teacher under Burk at San Francisco State Normal School, introduced Burk's plan into the Winnetka schools with particular adaptations.[3] The curriculum was divided into two parts: skills and knowledges to be mastered by all students, and opportunities for each child's development of self-expression. In the former, the time required varied from student to student, but each child achieved mastery. In the latter, the student's work was directed by his interests and abilities. Half the morning and half the afternoon were devoted to group and creative activities without set goals and tests, as the terms are understood today. The other half of the school day was organized so that each child would do a unit of work, check his answers, and go on to the next unit. Comparison of the Winnetka children with students from comparable schools using traditional methods are reported in the 24th N.S.S.E. Yearbook.

After standardized tests had been administered, the children were compared by grades, achievement groups, and by mental age groups in the various subject areas. The Winnetka children did better work on these tests than the children from comparable schools in every subject.

Washburne and Raths surveyed the success of the Winnetka children in high school in terms of scholastic achievement and participation in school activities.[5] A comparison was made between the Winnetka children and students from neighboring elementary school systems. Grade averages were computed for each of the four high school classes. Although there were not significant differences between the two groups of students in terms of I.Q. scores and chronological age, Washburne and Raths found that the Winnetka students were the only ones in the New Trier Township High School to achieve above the school average in all five major subject areas. In addition, the Winnetka children led in the proportion of student government offices held, and in participation in school organizations.

A few months after the inauguration of the Winnetka Plan, a form of individual instruction known as the Dalton Plan was introduced into a public high school in Dalton, Massachusetts. This was one of the first attempts at individual instruction on the secondary level.[6] This plan emphasized freedom, community living, and student responsibility. Under the Dalton Plan, freedom referred to the opportunity given the student to pursue an area of interest without interruption. Community living meant replacing grade rooms and teachers with subject matter laboratories and specialists. Several grades of students would have access to as many laboratories; living within the school was much the same as in the community outside. To develop student responsibility in the area of time budgeting, the work of each class was organized in the form of a job. It was then up to the student to budget his time in terms of his abilities and needs. The units were teacher constructed; they were called "contracts" since the student "contracted" to complete a pre-determined amount of work.

W. H. Thompson, experimentally tested the Dalton Plan at the Ohio State University Demonstration School.[7] Thompson compared the Ohio State school and Columbus, Ohio public schools. Batteries of standardized tests were given in all subjects at the beginning and end of an 8-week summer session. Mental age and achievement in reading, composition, spelling,and arithmetic were measured. The same procedure was followed during the next summer. Both times the Dalton groups were superior on the initial tests, but the gains did not favor either group. Thompson acknowledged, however, that the Dalton class size was small and there were daily visitors who could have distracted students from their work.

Different results were found by G. H. Mayer-Oakes.[8] Mayer-Oakes was particularly concerned with the effects of the Dalton Plan upon discipline problems, achievement on state tests, and students' maximum use of their time. Five participating classes were set up and contracts prepared. The students in these classes worked alone or in groups as they desired. They were required to spend at least one hour in a subject laboratory or room. Stnadardized tests were given after the twelfth and the twenty-eighth week. In addition, the scores on the state examination were also analyzed. Although there were no classes, and the number of students involved in the experiment was small (49), Mayer-Oakes concluded that the Dalton Plan improved the student's learning in a small high school because standardized tests showed that the Dalton students were above the standard median scores in all cases except algebra and English 11 and 12. In addition, he found that there was a gain of 25 per cent in the state test pass marks with 31 per cent more pass marks than the state average.

Abelson concentrated upon the effects of the Dalton Plan on the slow learner.[9] He computed the A. Q. (Accomplishment Quotient), a measure of achievement in relation to capacity, for 104 of 169 unselected students in the sixth grade at P. S. 39 in the Bronx, New York. Each of the 104 students had had a nimimum of one year of study under the Dalton system. *Stanford Achievement Tests* were given in all subject areas, and the *National Intelligence Test* and the *Haggerty Intelligence Test Delta 2* were given for I. Q. scores. In his analysis, Abelson divided the group of 104 students into three divisions: superior, average and dull. He found that the 16 dull pupils in the 6B-op class (a class for the slow learner) scored more points above their expected A. Q. than the superior students, and almost as many points above as the average students. This might have resulted from an underestimate of the expected A. Q. Similar results were observed in the 6A-op class although they were less marked. Abelson concluded that the Dalton Plan seemed to work better with the dull than with the bright student and almost as well as with the average student. He recognized the limitation of his small sample and possible defects in the computation of the A. Q.

A carefully controlled experiment was carried out by Esson and Cole.[10] 275 students from ten first-class North Dakota high schools were involved in this experiment. The control group, taught in the usual manner, contained 137 students as compared with the 138 students in the experimental group. Different teachers taught different groups. Both groups were given the same initial and final tests; the Gregory Tests on American History (Test III, Form A) and the Van Wagenen Reading Scale. For the analysis, the pupils were matched on the basis of school size, history comprehension test score and reading scale score. In all 87,pairs were formed, and in addition, schools were compared with schools as well as individual with individual. The investigators found that the differences between the individual matched pairs were in favor of the experimental group but these differences were not significant; the same was true for the group comparisons.

Schultz and Ohlsen tested the effectiveness of individualized instruction in providing remedial help to algebra students.[11] Actually, two experiments were carried out as Schultz and Ohlsen each taught a control and experimental group. There were slight gains favoring the experimental groups over the control groups although the differences were not significant. The investigators did point out, however, that the experimental group students did appear to have a more satisfying experience in that they appeared to be a "happier" group, they expressed appreciation for the opportunity of working at their own rate, they enjoyed the added responsibility, and there were some favorable changes in their attitudes towards mathematics. These observations were drawn from teacher-requested personal reactions written by the students.

Working with ninth grade students, Bernstein attempted to overcome arithmetic deficiencies by establishing a mathematics clinic which offered individual instruction to small groups of students.[12] Bernstein found that individualized instruction enabled students to accomplish twice as much work in less time than those students taught in large remedial groups. He also reported gains in the overall school adjustment of the students participating in the individualized classrooms, although this information was the result of observation.

There are several studies of classroom "climates" that have some, although indirect, relationship to the present study. Buswell, having examined the relationship between the social structure of the classroom and the academic success of pupils, reported that there was a relationship between achievement and social acceptance.[13] In the kindergarten, Buswell noted that the future achiever was not preferred to the future non-achiever in social relationships with peers. In the higher grades, however, it was generally true that academic success and social acceptance went hand in hand. This would seem to indicate that in order to enable all students to achieve maximally it is essential that social relationships be satisfactory.

Lewin, Lippitt, and White did a most interesting study varying the climate of boys' clubs from autocratic to democratic to laissez-faire.[14] Social relationships were perceptably influenced by the leader's behavior. Hostility and aggression were more prevalent in the autocratic group. There was a sharp rise in aggression when the autocrat left the room.

The two studies above contributed to plans for the activities of the experimental group of this study as did the work of Bovard; in one group student-to-student verbal interaction was fostered, in another, it was firmly limited.[15] Bovard concluded that the amount of social interaction in the classroom influenced students' feelings, perceptions and interpersonal relationships.

That other factors may influence students' feelings and perceptions was reported by Schilhab who studied the attitudes of 62 boys and 39 girls in first year algebra in the summer session of the Austin (Texas) public schools.[16] He found that attitudes were influenced by arithmetic achievement test scores, intelligence quotients, and high school grade averages. The same attitudes were not influenced by reading achievement test scores nor by family social class. A poor foundation in the fundamentals of mathematics and no realization of a practical value for algebra were associated with attitudes. Unfortunately, Schilhab limited his study to summer school students with past failures in mathematics; in addition, he did not mention the reliability and validity of the questionnaire used to determine student negative attitudes.

In summary, it may be said that while there have been many studies in which methods were related in some way or another to knowledge of individual differences, the experiment reported herein is the only one in which the experimental design was such as to permit replication with other personnel, a factor which will be mentioned again in the recommendations.

CHAPTER IV - PLAN

An informal pilot study in elementary algebra in which an attempt was made to find methods of individualizing instruction resulted in the development of a technique which seemed to show promise.[1] Because of the potential significance of this new approach, a rigorous experimental design was created to test the hypotheses that ninth grade mathematics pupils taught by individualized and traditional methods do not differ in achievement or attitude. Experience with the pilot project pointed up the need for materials which would enable students to perceive relationships and thus result in pupil-discovery of algebraic concepts. In addition, the nature of the planned experiment was such that the use of large numbers of students was essential; this fact, coupled with the change in roles required of students and teachers in the individualized method, necessitated an intensive training program for the participating teachers; therefore, a six-weeks'summer institute divided into two three-week periods was planned. During the first three weeks it was planned to develop perceptual materials and orient teachers to the individualized method. During the second three-week period a voluntary group of ninth grade students from the local area provided the teachers with an opportunity to experience teaching by the individualized method and to evaluate and revise newly created perceptual devices.

Experimental Design

Because one of the primary purposes of this investigation was the evaluation of a relatively new and untried method of individualized algebra instruction involving visual, auditory, and manipulative materials, a randomized block design with replications was chosen as the plan which would yield the most information concerning the relative effectiveness of the individualized and traditional methods. Differences in effectiveness between teachers were of no concern in this study; hence,these differences were assigned to block effects, each teacher being required to teach two ninth grade classes by each of two methods. Likewise, effects due to differences in schools were spread evenly through the methods comparison by balancing the design and completely confounding teacher and school effects.

The plan involved 36 ninth grade algebra classes, with a total of 904 pupils, taught by nine teachers in nine public schools, each teacher teaching two experimental and two control classes. Pupils were assigned by a random method to one of four classes in each school by school personnel (See Appendix A) and the teachers were instructed to assign classes to the two methods by randomization procedures. The exigencies of the school situations required in certain instances that the randomization of classes to methods could not be carried out as directed, but there is no reason to believe that the methods assignmemt should therefore be described as haphazard. In one instance, for example, the two classes which met in the same room had to be designated "experimental" since otherwise perceptual materials would have had to be carried from one floor of a school building to another. In a few schools changes in pupil schedules had to be made during the first week of school; effort was made by teachers to omit the scores of pupils who were added to the classes after the randomization was completed. An analysis of covariance using scores from four sub-tests of the Differential Aptitude Test (to be described shortly) as the independent control variable was utilized to increase the precision of the methods comparison.

Measures

The measures used in this experiment were: The Differential Aptitude Tests (DAT), a specially constructed Attitude Toward Mathematics Scale, and a Mathematics Achievement Test designed to reflect growth in three different aspects of mathematics learning--manipulation of symbols, understanding of relationships, and problem-solving skills.

The D.A.T. was used to obtain a measure of the ability of each student at the beginning of the experiment; although it measures capabilities in many areas, only those abilities that were germane to the subject matter of this study was tested.[2] Numerical Ability, Verbal Reasoning, Abstract Reasoning, and Space Relations tests were administered to all pupils in the study in September 1960. The scores of the separate sub-tests were combined into a single composite score which was used as the independent variable in adjusting the end of term achievement scores for initial ability (see Technical Supplement, page 28). These tests were administered in 115 minutes of actual testing; each test required thirty minutes except for the Abstract Reasoning test which took twenty-five minutes.

In his review of these tests, D. E. Super states, "The authors have done such a thorough and technically satisfactory job that a reviewer finds it hard to make himself appear sufficiently critical."[3]

The correlations of the four sub-tests with mathematics grades of ninth year algebra students as derived from the D.A.T. manual are:

Verbal Reasoning	.32
Numerical Ability	.51
Abstract Reasoning	.37
Spatial Relationships	.24

The median reliability coefficients and standard errors of measurement based on the ninth grade sample are:

	Coefficient	S.E.
Verbal Reasoning	.89	2.7
Numerical Ability	.85	2.9
Abstract Reasoning	.87	3.7
Spatial Relationships	.91	6.7

The standard errors are in terms of raw scores.

The Attitude Toward Mathematics Scale (See Appendix B) was used to obtain a measure of the attitude of each student toward mathematics.[4] This test contains 50 statements with which the student is asked to agree or disagree; the score is determined by the number of statements in which a student has indicated a desirable attitude toward mathematics. The test requires approximately twenty minutes to administer, although a student is permitted all the time he may need.

The face validity of the instrument was established by submitting it to a jury consisting of a mathematics educator, a psychologist, and a statistician.

To establish reliability, seventy items were originally constructed. On administration to 125 students in the ninth grade of a New York City junior high school and a similar school in a suburban area, application of the Kuder-Richardson Formula No. 20, yielded reliability coefficients varying from .81 to .90. The scale was then revised by eliminating items that were ambiguous and those that were relatively non-discriminating. The revised inventory was again administered to 124 different students in the aforementioned schools, and on this occasion the Kuder-Richardson Formula No. 20 yielded a reliability coefficient of .85.

Every effort was made to find a standardized test which would adequately sample the content of the experimental curricula but because none of the available standardized tests reflected all three aspects of ninth year algebra achievement (manipulation of symbols, understanding of constructs and relationships, and problem-solving skills) it was necessary to construct a 117 item test (see Appendix C) designed to reflect growth in the three specific aspects mentioned above. No opportunity was available for standardization and lack of time prevented more than establishment of content validity. Items were, however, designed to measure learning in all areas which the teachers reported had been studied by individual children.

Schedule

Briefly the schedule for the development of the project was:

Spring 1960 -	Recruitment of teachers and ninth-grade students
Summer 1960 -	Institute - 3 weeks orientation and perceptual-device construction 3 weeks practicum
Sept. 1960 -	Classroom phase of Experiment
Sept-Oct.1960	Preliminary testing
June 1961 -	Final testing

Recruitment of teachers to participate in the experiment was done in the spring semester 1960. An effort was made to secure teachers who indicated no particular bias toward either of the methods which were to be evaluated and precautions were taken in order to reduce potential "Hawthorne effect."

During the summer of 1960, a six weeks' workshop was held at Queens College. For the first three weeks the participating teachers, three undergraduate students, and the investigators, together with consultants who visited from time to time, discussed the technique and role-played "individualization." In addition, everyone assisted in construction of perceptual devices. During the second three weeks, 25 children from the Queens College neighborhood constituted a class in which the participating teachers practiced the techniques which had been discussed and role-played, and tried out the perceptual devices constructed during the first three weeks.

During the school year 1960-61 the experiment itself was conducted and during the summer of 1961 the data analyzed. It had originally been planned to test the hypothesis that average algebra achievement measured one year after the end of the experimental period would not differ for classes taught by individualized and traditional methods by re-testing in June 1962, but the results of the 1961 analysis of data seemed not to warrant the expense involved and the experiment was concluded in June 1961.

Summary

A pilot study resulted in the development of a technique which seemed to show promise of individualizing instruction in accordance with student individual differences.[5] In order to test the efficacy of the method, an experiment based on a randomized block design was performed. In preparation for actual experimentation perceptual devices were constructed and teachers oriented to the individualized method. Instruments used to test the hypotheses were: The Differential Aptitude Test, an Attitude Toward Mathematics Scale, and a Mathematics Achievement Test (see Appendices B and C).

CHAPTER V - PROCEDURE

Recruitment

As indicated in the schedule, participating teachers for the experiment were recruited in the spring of 1960. Original plans called for five public school teachers from New York City and five from Long Island so that both suburban and city populations would be sampled. Accordingly, the Associate Superintendent for Junior High Schools in New York City recommended five algebra teachers from his division. Another five teachers, one of whom dropped out before the experiment was well underway, were accepted on the basis of personal application and the recommendation of department chairmen or principals. Personal interviews were conducted in an attempt to select teachers who indicated no particular bias toward either of the methods which were to be evaluated.

From the ninth grades of junior high schools near Queens College (Flushing, N.Y.) and from the classes of the participating teachers, twenty-five pupils were recruited to attend a three-weeks' summer session at the college. The purpose of this activity was to provide a class for the participating teachers in which they could practice individualized instruction, try out perceptual devices, and provide a review of ninth grade algebra for a group of students who were about to enter tenth grade.

Early in June a two-day conference was held at which time the consultants, Professors Morris Kline, Warwick Sawyer, and Leo Zippin discussed their ideas regarding perceptual devices with the investigators and some of the participating teachers.

Summer Session

During the summer of 1960, a six-weeks' workshop was held at Queens College every day, five days a week, from 9:00 a.m. until 4:00 p.m. The usual procedure was to spend the early part of the morning discussing the psychological foundations of individualized instruction and the methodology in terms of the classes the teachers would meet in the coming semester. At times, the Project Director, Mr. Herbert Fremont, who had individualized instruction in his algebra classes for three years, would demonstrate by role-playing. The teachers would assume the roles of ninth graders and the Project Director would "teach". In most instances the lunch hour was devoted to discussion of the introduction of the technique to the experimental classes.

At other times one of the consultants might meet with the group to discuss the perceptual devices which were needed. At another time, the chief consultant, Professor Harold E. Mitzel, instructed the teachers in the technique of administering standardized tests and answered questions regarding the statistical treatment of the data that were to be gathered.

Shortly after these verbal sessions everyone gathered in the "shop" where the teachers usually worked in pairs to construct the perceptual devices which are listed in Appendix D. In order to provide the reader with some insight into the construction and use of the devices, a brief description of one is included immediately below and in addition Appendix F contains a complete description of one and brief explanations with illustrative drawings of several others.

The Hill (Appendix F) is merely a wooden plane which can be inclined at a variety of angles and which is bounded by "bannisters" which serve no other purpose than keeping a ball within bounds. Attached to the plane is a pendulum which keeps time in seconds (1 swing of a 39" string per second). A small rubber ball - and a bucket to catch it complete the equipment. Directions to the students enable them to discover the solution to certain motion problems in algebra.

Lack of space prevents a full explanation of all the perceptual devices developed.* During the three-weeks' summer session for ninth grade pupils which was part of the Institute, all completed perceptual devices were tested; as a consequence, revisions and improvements were made. In addition, each teacher took charge of the class for one day; it was hoped that this experience would help the teachers to conduct their experimental classes with some degree of confidence.

The School Year

During the school year 1960-1961, the classroom phase of the experiment was conducted. As stated previously, the Differential Aptitude Test and the Attitude Toward Mathematics Scale were administered by the teachers in September 1960. Thereafter the experiment proceeded as planned. The investigators visited each school every five weeks during the first half of the school year. In addition, a newsletter was published (Appendix G) and at monthly meetings the participating teachers exchanged information and suggestions. Consultants visited the schools from time to time. These activities were attempts to build confidence in the teachers who were experiencing the individualized approach for the first time in their own classes.

At mid-year all the ninth grade pupils in the city schools were required to take a city-wide examination; it was decided by the investigators that no exception should be made of the experimental classes. Copies of the examination were made available to the four schools on Long Island as well. The New York City teachers were required to send results of these examinations to the office of the Coordinator of Junior High School Mathematics; all teachers were requested to send results to the Research Unit of the Division of Teacher Education for preliminary analysis. Despite previous discussion about the possibility that the experimental groups would not do well on this particular examination, since the pupils had participated in setting their own goals and therefore were not necessarily following the curriculum in the prescribed order, it became obvious that the results created considerable anxiety. Teachers verbalized this anxiety to the investigators and, in a number of instances, they began to use traditional procedures in the experimental groups. In particular, this took the form of eliminating individual goals in favor of group goals chosen by the teachers in order to enable pupils to "catch up" with content they had not covered before this examination.

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* For further information on the devices write: MILE, Education Department, Queens College, Flushing 67, New York.

At this time the Chief Investigator was relieved of certain teaching requirements in order to visit each city school once a week and the Project Director increased the number of visits to schools on Long Island. Subsequent observation made clear that in some schools individualization procedures had given way to traditional, culminating in intense concentration, about a month before school closed, on preparation for the city-wide final examinations. As a result of these observations the investigators decided to give the final experimental examinations sufficiently early to avoid contamination of results by intensive coaching.

Because of difficulties connected with collecting and forwarding the tests administered in September, 1960, it was decided that the final achievement examination should be administered as well as scored by personnel of the research office of the City University's Division of Teacher Education. In order to avoid taking any more school time than absolutely necessary and because there was adequate free time in the last week of school, teachers were asked to administer the Attitude Toward Mathematics Scale at that time. Six of the nine teachers did so.

CHAPTER VI - RESULTS

A complete analysis of the data is included in the Technical Supplement. Table 1 summarizes the means from nine schools on both achievement and aptitude tests for the major methods comparison.

Table 1

Means of Algebra Achievement Measures (Raw Scores) and of Learning Aptitude (T scores) Showing Differences Between Ninth-Grade Experimental and Control Groups from Nine Junior High Schools.

Nine Schools	Math Achievement Raw Scores	Learning Aptitude T Scores	N
Experimental (Individualized)	55.49	50.52	450
Control (Traditional)	58.60	50.49	454
	-3.11	+0.03	904

There are wide differences between schools in both Mathematics Achievement and in Learning Aptitudes but because these differences are not of primary concern they have been eliminated from the final analysis. As can be seen by inspection of Table 1, the achievement scores of the sample of control classes were higher than those of the experimental. The observed differences do not reach the five percent significance level and it is therefore logical to accept the null hypothesis and to conclude that the differences are consistent with chance findings. The closeness of the decision, as shown in the Technical Supplement clearly indicates the need for further study. Such study should provide for adequate attention to all relevant variables which influence the mean despite the fact that this may require the development of techniques to facilitate analysis of these variables without loss of precision.

Perhaps the most obvious illustration of this need was the not-uncommon situation in the experimental classes in which a number of students finished about one-half the work of the term. Their final examination marks were very low although their mastery of the limited amount finished was thorough; despite the fact that both teachers and students experienced success in the situation, their effect on the mean for the experimental classes as a whole was a depressing one.

The Attitude Toward Mathematics Scale presents a more difficult problem of analysis. Of the nine participating schools, six administered the end-of-year inventory, yielding pre and post scores for approximately 588 pupils, as compared with the 904 students involved in the achievement analysis. One puzzling result, as is shown in Table 2, was the fact that virtually all mean attitude scores went down on the post-test.

Table 2

Mean Scores on the Attitude Toward Mathematics Scale

	Mean Scores			
	9/60	6/61	Change	N
Experimental (Individualized)	37.07	34.69	-2.38	288
Control (Traditional)	36.93	34.65	-2.28	300
All Pupils	37.00	34.67	-2.33	588

There is a significant drop in mean attitude scores of pupils taught by both methods from the beginning to the end of the term. As some statistical wit remarked on seeing the data, "If you want kids to like Algebra, keep them away from math teachers."

To evaluate the figures objectively it is necessary to note that there were significantly different changes in means from school to school (See Table 7, Technical Supplement), indicating that teacher-differences might have affected attitudes. Again, however, the differences between Experimental and Control Groups are large and,in this case, significant at the five per cent level. The data clearly indicate that, although the pupils as a group scored lower on the attitude scale in June than they had in September, those in the Experimental classes scored lower than those in the Control. Possible causative factors will be mentioned in the section following.

Discussion

Because this study was designed to compare change in both knowledge and attitude of pupils taught by two different methods, this discussion will focus first on achievement, second on attitude and third on the possible relationship between achievement and attitude changes.

As noted in the Technical Supplement, the failure of the F-ratio in the adjusted analysis (Technical Supplement, Table 3) to meet the five per cent significance level led to acceptance of the null hypothesis and the conclusion that the observed differences between adjusted means for methods are consistent with chance findings. It would, however, be premature to conclude that the two methods are equally effective in promoting the learning of ninth grade algebra. The fact that the F-ratio of 4.14 justified acceptance of the null hypothesis whereas an F-ratio of 4.24 would have led to rejection, raised the question of the possibility of a Type II error. On the other hand, observations during the year and interviews with participants led the investigators to make decisions which may have influenced the final results in such a way as to minimize the differences between Experimental and Control groups. For example, when two teachers discontinued individualization in the Experimental classes after mid-year examinations, the schools should have been dropped from the study. On the basis of observation and discussion with the teachers, it was assumed that individualization would be restored when the teachers felt confident enough to do so. This was an error on the part of the investigators.

While observation and interview are unreliable tools, it is important to note that they did have a bearing on decisions made by the investigators and therefore contributed to whatever bias might have affected students' scores. It has long been known that two observers with slight differences in perception will make fractionally different observations and thus affect the outcome of an experiment. In the physical sciences instruments have been devised which make the "human equation" obsolete; in education such exactitude has not been achieved. Listed below, therefore, are points of information (based on observations and interviews) which may or may not have had some influence on student achievement.

1. In an effort to avoid the "Hawthorne Effect" the investigators emphasized in the first interview with each teacher that preference for one method over another would adversely affect the experiment. The teacher's verbal statement of no bias was accepted at face value primarily because of (1) the difficulty of comparing an unknown (individualized) method with a known (traditional) one; (2) the lack of a technique which would reveal unconscious preference; and (3) the knowledge that for a period time (until the unknown becomes known) there is a tendency for human beings to prefer the known, familiar way of doing anything over the unknown and unfamiliar. During the first three weeks of the summer institute, it became clear that there were anxieties aroused by the prospect of "pupil-teacher planning" which the experimental method required, and which none of the teachers had done before. Two comments on the last day of summer session might reveal the attitudes of the teachers just as the classroom phase of the experiment was about to begin. One said, "I didn't think much of this idea (individualized instruction) when I first heard of it, but now I think there is something (worthwhile) to it." And another said, "I disagree with most of your principles (individualized instruction) but I give you my word that I'll do my best on the experiment."

2. Seven of the nine teachers were requested to volunteer for the experiment by their superiors in the school hierarchy. This pressure appeared along a continuum from a mild request (which the teacher seemed eager to grant) to a statement to the effect that failure to participate would result in an undesirable teaching assignment. The resultant anxiety was noted during the summer institute but because the investigators did not know of the pressure until the term was more than half over, it was not dissipated effectively. At this writing, the investigators are unable to indicate how the intensity of such pressure could have been determined in an initial interview, or at any time before there was sufficient confidence built up between the investigators and the teachers to enable everyone to be completely frank in discussion. There is, of course, no evidence to substantiate any claim that these pressures on the teachers had any effect on the outcome of the experiment and the two teachers whose participation was truly voluntary were two whose control groups had better achievement scores than their experimental groups. Nevertheless, there are unknown qualitative factors in individual reactions to pressure which cannot be dismissed lightly.

3. As stated previously, in two schools teachers discontinued individualized instruction in the experimental groups after the midyear. They made no attempt to conceal their change of plans but did give the investigators reason to believe (perhaps"hope"is a better word) that once their difficulties had been resolved, individualization would be re-instituted. In another school, the investigators inadvertently discovered that the experimental class

was taught in the same manner as the control class except during the one day a week when the class was observed by the chief investigator. This was discussed with the teacher and with the school principal, but no decision was made to eliminate the school from the experiment, probably because the investigators feared the loss of subjects, but also because a few children were benefiting from the use of certain materials which were available only to the experimental groups. These three schools present examples of the human factors which influence decisions; all three replications should have been dropped. Failure to do so was an error on the part of the investigators.

4. Some had difficulty because of what they perceived to be the magnitude of the adjustment required to keep the methods separate. One teacher reportedly used the materials which were designated for the experimental group only, with all four classes. This was not known to the investigators until after the experiment was finished.

While it is not possible to distinguish factors which might influence achievement from those which might influence attitudes, the previous four statements are indicative of factors which might have had some effect on pupil achievement. They conceivably might have had some influence on pupil attitudes as well. In addition, two other factors might well have affected student attitudes.

1) One teacher had difficulty in applying the individualized method because the school principal objected to the noise, and his frequent visits to the room to urge greater quiet made "algebra period" one of the less pleasant hours of the day for the children. This type of difficulty may have influenced the achievement scores, but it seems to the investigators to have had a greater effect on morale and thus the effects would be more likely to show up on attitude scores.

2) Pupils in the experimental classes frequently verbalized a feeling that they weren't being helped in their learning by the teacher; they said that they had "to teach themselves" or that they "had no teacher." These expressions, also used by the teachers, would seem to indicate a basic misconception regarding the experimental method which would be conducive to developing an unpleasant attitude toward mathematics. The respective responsibilities of teachers and students were evidently not well enough defined during the institute.

In addition to these human factors which may, or may not, have had some effect on the results, it is necessary to consider whether the drop in scores reflect attitudes characteristic of the majority of ninth graders toward all their subjects at the end of the school year or whether it was unique to these algebra classes.

The attitude scores were, because of the failure of certain teachers to return results, based on smaller numbers than the achievement tests and the question must therefore be raised whether the inclusion of twelve classes would have influenced the results. One needs to know, also, if changes occur in teachers' attitudes toward the end of the year and if there are corresponding student changes. Since these things are not known, it is important to avoid trying to determine causal relationships. Other reasons for withholding judgment are: the ease with which scores on such instruments are skewed, the pressures put upon graduating ninth graders during the last week of school, and the fact that, as far as can be determined by the investigators and school administrators, these teachers, on the whole, were better than average.

Conclusions

It must be said, in conclusion, that certain factors which became obvious as a result of observation or conversation seem to be important, although there is no factual evidence to support the claim of personal judgment that they are. A sincere effort was made to secure the cooperation of teachers who had a healthy skepticism; i.e., open-mindedness regarding individualization. By the time the ninth grade summer algebra class had begun, there was subjective evidence that few of the teachers were unbiased. Only two teachers can be so described; another two (one of whom dropped later)"believed in" individualization; the other six seemed to the experimenters to be convinced that the experimental method was impractical and while they were willing to participate, they were quite sure that the teaching methods they had been using, and planned to continue to use in their control classes, were superior to individualization. As a result of this experience, it is logical to conclude that the teachers' attitudes toward the two methods were not entirely unbiased at the beginning of the experiment. It seems a serious weakness of many research studies on methods that no measure of teacher-attitudes is given at the beginning and end since there is some evidence in this experiment that teachers' attitudes have much more widespread effect than might be supposed.

Since three of the teachers had disciplinary problems in the experimental groups (one of these was a teacher of many years experience who had had no trouble with discipline previously and who was considered an excellent teacher by his principal) we must conclude that it is difficult for some teachers to master the "science" of individualization and that old habits cannot be replaced by new in one six-weeks' summer session. Almost all the teachers reported difficulty in individualization, and observation confirmed the conclusion that in almost every school one or more of the conditions described above as characteristic of individualization (see Fig. I, page 7) was inoperative at one or more periods of time. One must also conclude that the experiment was not an adequate measure of two different methods in view of the evidence that (1) traditional techniques were used in some of the experimental classes, and (2) perceptual devices designed for the experimental classes were used in at least one of the traditional classes.

A further observation is that the exactitude of research methods create difficulties for teachers, principals and school administrators in general. The randomization procedures were exceedingly difficult to follow precisely, and, in some instances, school authorities, far removed from the experiment, could not be expected to have more than a vague and general sense of responsibility for the research. It must be added, however, that this conclusion is a statement of fact and not an indication of failure to be cooperative; in fact, there was cooperation against odds--principals who had to spend hours explaining the experiment to parents, teachers who spent hours on perceptual devices--but the day-to-day demands of the experiment were too much to ask of the teachers. In fact, merely becoming familiar with all the perceptual devices required more time than teachers could give.

Despite the many large-scale research projects which are reported in the literature every year, the experience of the present investigators raises some doubt about the possibility of conducting honest, scientific, time-consuming research in field situations. It may be that laboratory settings are the only places where sufficient controls can be maintained for the course of a precise experiment.

In short, then, we have found that:

1. Differences between achievement scores of pupils taught by individualized and traditional methods were not significantly different from what would be expected by chance.

2. Pupils in control and experimental classes had scores significantly lower at the end of the year than they had at the beginning of the term on an Attitude Toward Mathematics Scale.

3. Teachers, principals and administrators in this study found it difficult to fulfill all the requirements for the experimental design adequately. As a result, it is possible that many small violations of the design requirements combined to skew the results in the direction of the known over the unknown, whatever that might be.

CHAPTER VII - RECOMMENDATIONS

Because acceptance of the null hypothesis leaves many unanswered questions it is necessary to recommend that this study be repeated with certain changes. These are:

1. Select only those teachers for the experiment who are competent in both methods and prefer neither. This would require acceptance of the two teacher roles and a minimum of one year's experience utilizing both methods before the experiment began.
2. Develop techniques which would permit analysis of individual cases in large scale experiments. Since computation of a mean results in absolute anonymity of all the contributing elements, there is a paradox in the use of a measure of central tendency to test hypotheses founded on the assumption that differences between individuals should be magnified rather than minimized by the educational process.
3. Develop meaningful operational definitions of variables that will reflect all of the educational objectives inherent in instruction.
4. Involve the teachers as researchers. Some teachers are basically disinterested in research and there is a tendency to view the researchers' requests as mere pedantry; everyone involved in the research should be co-equal members of the research team.
5. Include research on retention. Since there is some evidence in a study by Calvin that retention in individualized instruction does not regress toward chance differences, it is important that studies be made of retention regardless of initial differences on immediate achievement scores.[1]
6. Determine pre and post experiment attitudes of teachers toward children and toward the two methods. Because of the teachers' psychological sophistication it may be necessary to explore the possible use of projective techniques.
7. Measure the degree of individualization in each class.
8. Clarify teacher perception of the term "student responsibility" as used in the experiment. One of the observations (see page 23) of the investigators was that attitudes were influenced by student and teacher feelings to the effect that the students were required to learn "on their own." This perception of the term "student responsibility" appears to have influenced experimental results.

It is also recommended that continuous research in methods of individualization be conducted. There are, in fact, many studies in progress by persons who are interested in illustrating the individualization potentialities in programmed learning. In these studies, students using the same program have learned at varying rates; there is need to know whether variation in the form and texture of the perceptual materials used; i.e., films, tapes, manipulative devices, etc., will also influence learning. While it is frequently assumed that interpersonal relationships influence learning, little is actually known how this occurs or to what degree. Most studies of personal interaction have as their purpose the understanding of interpersonal dynamics; it is important that the relationship between human interactions and learning be understood.

In summary, it is recommended that the search for better methods of individualization be continued and that student attitudes toward all subjects be studied further. The Attitude Toward Mathematics Scale should itself be an object of study and different forms for different subject areas should be developed. It is important to be sure that The Attitude Toward Mathematics Scale is a tester of attitudes and not merely a reflection of September enthusiasm and June weariness. Similarly, the Mathematics Achievement Test should be refined on the basis of findings regarding validity and reliability. Finally, it is recommended that this study be replicated under conditions which would permit absolute fidelity to the original experimental design.

FOOTNOTES

CHAPTER I

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CHAPTER II

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TECHNICAL SUPPLEMENT

Harold E. Mitzel

We are indebted to Professor Donald M. Medley for painstaking guidance in the intricacies of the analysis of variance and covariance designs used in this study. Mr. Stephen Margulis and Miss Helen Lee were responsible for the day-to-day processing of the data. They bore the brunt of the frustrations involved in computer failure and in the correction of errors.

This study is essentially a completely balancee randomized block experiment involving 36 classes which received instruction during 1960-61 in ninth-year algebra in 9 junior high schools in the New York Metropolitan area. Two methods, an experimental individualized approach and a traditional whole-group method, were employed by each of nine teachers in each of two classes. Randomization for the validation of experimental error stems from two sources. First, the ninth-grade pupils in any given school who enrolled for the first course in algebra were assigned by a random method (method of remainders from table of random numbers) to one of four classes. (See Appendix A). Second, the method to be used by the teacher was assigned to the particular class in random fashion. The latter is particularly important in spreading the influence of time-of-day effects equally between the two methods.

Variables of Study

Because this study was designed to compare the change in knowledge and attitude on the part of pupils who were taught by two different methods of instruction, four variables were employed in covariance and analysis of variance designs.

The two dependent variables were pupil achievement in algebra at the end of the ninth-year as measured by a specially constructed 117-item examination. The achievement test was designed to reflect three aspects of ninth-year mathematics growth; manipulation of symbols, understanding of constructs and relationships, and problem-solving skills. The other dependent variable was the end-of-year score of each pupil on a 50-item Attitude Toward Mathematics Scale. Details regarding the construction of these measures are given in the body of the report.

The independent variable used as a covariant term in the analysis of the achievement data was a composite T-score based on Form A of four sub-tests of the Differential Aptitude Tests. The four sub-tests: Verbal Reasoning, Numerical Ability, Abstract Reasoning and Space Relations were weighted equally in arriving at the composite T-score. The analyses described in this supplement are divided into two parts, one concerned with the achievement of pupils in ninth-year algebra; the other concerned with changes in attitude toward mathematics of ninth-grade pupils during the course of instruction with two different methods.

The Achievement Analysis

The dependent variable for this analysis is the total raw score on the comprehensive achievement examination which was administered to the 36 classes during the last week of May 1961. The independent covariant variable, designed to control the experimental data for differences in learning aptitude, was a composite standard score based on four subtests of the Differential Aptitude Tests. This is a mixed-model design in which method is considered to be a fixed effect and schools, classes and pupils are random variables.

Table I

Unadjusted Means* of Aptitude (T-Scores), Achievement Measures (Raw Scores) and Differences between Ninth-Grade Experimental and Control Groups in Nine Junior High Schools

Schools (and Teachers)		(Positive differences listed as favoring the Experimental Method)		
		Algebra Achievement (Dependent Variable)	Learning Aptitude¶ (Independent Variable)	N
1	Experimental (Individualized)	60.20	55.51	45
	Control (Traditional)	64.83	56.83	48
	Difference	-4.63	-1.32	
2	Experimental (Individualized)	42.30	44.51	43
	Control (Traditional)	46.86	43.72	43
	Difference	-4.56	+0.79	
3	Experimental (Individualized)	45.03	47.49	37
	Control (Traditional)	48.22	50.04	45
	Difference	-3.19	-2.55	
4	Experimental (Individualized)	52.61	50.57	61
	Control (Traditional)	62.03	50.59	61
	Difference	-9.42	-0.02	
5	Experimental (Individualized)	64.77	56.85	47
	Control (Traditional)	69.45	56.83	40
	Difference	-4.68	+0.03	
6	Experimental (Individualized)	42.40	43.68	50
	Control (Traditional)	47.24	45.52	58
	Difference	-4.84	-1.84	
7	Experimental (Individualized)	66.76	54.43	58
	Control (Traditional)	65.93	50.35	46
	Difference	+0.83	+4.08	
8	Experimental (Individualized)	63.33	52.02	54
	Control (Traditional)	66.88	53.10	60
	Difference	-3.55	-1.08	
9	Experimental (Individualized)	56.58	43.05	55
	Control (Traditional)	55.83	48.14	53
	Difference	+0.75	-5.09	
All Schools	Experimental (Individualized)	55.49	50.52	450
	Control (Traditional)	58.60	50.49	454
	Difference	-3.11	+0.03	

---

*Each mean is based on the combined scores of two classes of pupils who were taught by one teacher using the same methods. The classes met at different periods.

¶ The value of the variate is the composite Y-Score on four sub-tests of the Differential Aptitude Tests.

Table 1 shows a school by school comparison of the unadjusted means for the two methods on the dependent (Algebra Achievement) and independent (Learning Aptitude) variables with the differences between the two groups. Table 2 presents the basic sums of squares and cross-products in deviation form from which the analysis of variance was accomplished. Note should be made of the fact that the obtained dependent variable sum of squares for methods and the deviation cross-products for methods were modified to take the unequal frequencies into account. The procedure suggested by Kempthorne (1952) was used to remove the bias due to unequal cell frequencies from the methods comparison.

Table 2

Deviation sums of Squares and Cross-Products for Algebra Achievement and Aptitude Data Based on 904 Ninth-Grade Pupils in 36 Classes in 9 New York Metropolitan Area Junior High Schools, 1960-61.

Source of Variation	Degrees of Freedom	Sum of Squares		Sum of Cross-Products
		Algebra Achievement y^2	Learning Aptitude x^2	xy
Methods	1	3,204*	0	225*
Schools (Block Effects)	8	65,773	15,373	28,370
Interaction, Method X Schools	8	2,153*	739	443*
Classes Within Methods and Schools (Replications)	18	31,426	2,660	8,029
Pupils Within Classes	868	168,539	29,642	40,648
TOTAL	903	271,095	48,414	77,715

---

*Modified to remove effects of unequal cell frequencies in classes taught by different methods.

The first step in the process of testing whether or not the observed difference between the adjusted means of experimental and control groups is consistent with chance findings requires the isolation of an appropriate error term.

Table 3 shows how the method-by-school interaction term was tested against the replication-within-method-and-school term.

The power of the covariance adjustment is shown in the reduction of the classes subtotal sum of squares in Table 3 from 102,556 to 29,364. The F-ratio of 1.56, being considerably less than the rule-of-thumb 2.00, suggests the desirability of pooling the methods-by-school interaction with the basic error term (replications) to form a new residual based on 26 degrees of freedom in the original analysis. There are, of course, wide differences between schools both in Mathematics Achievement and in Learning Aptitude. However, because these differences are not of primary concern, being in nature of block effects, they have been eliminated from the final analysis of variance and covariance. Table 4 shows the analysis of the variation due to methods means tested against the pooled residual error term.

The failure of the F-ratio in the adjusted analysis to meet the five per cent significance level leads us to accept the null hypothesis and to conclude that the observed differences between adjusted means for methods are consistent with chance findings. There is, however, some discomfort with the findings because of the closeness of the decision. It would be erroneous to conclude that the two methods are equally effective in promoting achievement in ninth-year algebra as measured in this study. Further study is clearly indicated under carefully controlled conditions.

The Attitude Analysis

Only six of the nine original schools repeated the attitude inventory at the end of the school year. Pre-term and post-term scores were isolated for approximately 560 pupils. However, in order to meet the condition of equal cell frequencies which simplifies the analysis of variance procedures in a nestled three-way design, data from all classes were limited to 16 pupils. Cases were withdrawn by a random procedure until each of the 24 classes in the analysis contained exactly 18 pupils, each with beginning and end-of-term attitude scores.

Table 5 shows the design of the analysis of variance with the obtained values for the mean squares and the expected mean squares which guide the choice of the appropriate error term for testing hypotheses. In this analysis pupils, classes, schools and change are considered to be random variables; method is a fixed effect.

For convenience of interpretation we divide Table 6 into two parts, one part (above the dotted line) shows all of the main effects and interactions in which change is not involved. The second part presents the main effect for change and all of the interactions in which change is involved. Non-significant interactions were combined with the residual in those instances where it was warranted. The data clearly show a significant drop in mean attitude scores of pupils taught by both methods from the beginning to the end of the term. In addition, the importance of the change-by-school interaction lies in the fact that there were significantly differing changes in means from school to school. The significant effects, change (disregarding schools and methods) and change-by-schools (disregarding methods) are shown in the form of attitude score means in Table 7. The largest drop in mean score (almost five points) was shown by the four classes in School 8, while a small gain (almost one-half point) in Attitude Toward Mathematics was registered by the pupils in School 7. The body of the report presents an interpretative discussion of these results.

Table 3

Analysis of Variance and Covariance of Achievement and Aptitude Data From Table 2 Showing Test of Significance for Methods-By-Schools Interaction

Source	Original Analysis				Aptitude Sum of Squares	Cross-Products	Adjusted Analysis				
	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio			Degrees of Freedom	Sum of Squares	Mean Square	F-ratio	Hypothesis Decision
Schools (Blocks)	8	65,773			15,373	28,370	9	16,909			
Methods	1	3,204			0	225					
Methods X Schools, Interaction	8	2,153	269	<1.00	739	443	8	5,264	658	1.56[¶]	Accept P>.05
Replications or Classes within Method and School	18	31,426	1,746		2,660	8,029	17	7,191	423		
Sub-Total Variation Among Classes	35	102,556					34	29,364			

[¶] $F_{8,17}$ = 2.55 at five per cent level of significance.

Table 4

Analysis of Variance and Covariance of Achievement and Aptitude Data From Table 2
Showing Test of Significance for Methods With Pooled Residual as Estimate of Error

	Original Analysis				Aptitude Sum of Squares	Cross-Products	Adjusted Analysis				
Source	Degrees of Freedom	Sum of Squares	Mean Square	F-ratio			Degrees of Freedom	Sum of Squares	Mean Square	F-ratio	Hypothesis Decision
Methods	1	3,204	3,204	2.48 (P>.05)	0	225	1	2,067	2,067	4.14¶	Accept P>.05
Pooled Residual	26	33,579	1,292		3,399	8,472	25	12,463	499		
Sub-Total	27	36,783			3,399	8,697	26	14,530			

¶ $F_{1,25}$ = 4.24 at five per cent level of significance.

Table 5

Expected and Observed Mean Squares for Analysis of Change in Attitude Scores for 432 Ninth-Grade Pupils in 24 Classes Measured at Beginning and at End of Term in Six New York Metropolitan Area Schools, 1960-61

urce of Variation	Degrees of Freedom	Sum of Squares	Observed Mean Square	Expected Mean Square
hods of Instruction	1	48	48	$\sigma^2 + 2\sigma_P{}^2 + 72\sigma_{MS}{}^2 + 36\sigma_R{}^2 + 432\ \theta_M{}^2$
ools (S)	5	1009	202	$\sigma^2 + 2\sigma_P{}^2 + 36\sigma_R{}^2 + 144\sigma_S{}^2$
sses within Method (R)	12	1406	117	$\sigma^2 + 2\sigma_P{}^2 + 36\sigma_R{}^2$
eraction, Method X School	5	551	110	$\sigma^2 + 2\sigma_P{}^2 + 72\sigma_{MS}{}^2$
ils (P)	408	42515	104	$\sigma^2 + 2\sigma_P{}^2$
nge, Beginning to nd of Term (C)	1	1219	1219	$\sigma^2 + 18\sigma_{RC}{}^2 + 72\sigma_{CS}{}^2 + 432\theta_C{}^2$
eraction, Change X School	5	723	145	$\sigma^2 + 18\sigma_{RC}{}^2 + 72\sigma_{CS}{}^2$
eraction, Change X Classes	12	606	50	$\sigma^2 + 18\sigma_{RC}{}^2$
eraction, Change X Method	1	19	19	$\sigma^2 + 18\sigma_{MC}{}^2 + 36\sigma_{MRS}{}^2 + 216\theta_{MC}{}^2$
eraction, Change X chool X Method	5	379	76	$\sigma^2 + 36\sigma_{MRS}{}^2$
nge Residual	408	15532	38	σ^2
al	863	64007		

Table 6

Condensed Analysis of Variance Showing Tests of Hypotheses Concerning Change in Attitude Scores for 432 Ninth-Grade Pupils

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square	F	Error Term for Testing Hypothesis	Hypothesis
Methods	1	48	48	-	Not Tested	
Schools	5	1009	202	1.92	Pooled Pupils, Classes Within Method & Method-School Interaction	$P > .05$ Accept
Pooled: Pupils, Classes within Method & Method-School Interaction	425	44,472	105	-		
Change Beginning to End of Term	1	1,219	1,219	8.41	Change and School Interaction	$P < .05$ Reject
Interaction, Change and School	5	723	145	3.82	Pooled Residual	$P < .05$ Reject
Interaction, Change and Method	1	19	19	-	Not Tested	
Interaction, Change, Method and School	5	379	76	2.00	Pooled Residual	$P > .05$ Accept
Pooled Change Residual (Including Interaction, Change and Classes)	420	16,138	38			
TOTAL	863	64,007				

Table 7

Mean Attitude Toward Mathematics Scores of 432 Ninth-Grade Pupils in Six Metropolitan Area Schools Measured at the Beginning and At the End of the 1960-61 School Year

	Mean Attitude Scores		
	Beginning of Year N-72	End of Year N-72	Change
School 1	37.78	36.03	-1.75
School 4	37.93	34.38	-3.55
School 6	38.04	37.12	-0.92
School 7	34.99	35.47	+0.48
School 8	38.90	33.96	-4.94
School 9	36.07	32.50	-3.57
Grand Mean	37.28	34.91	-2.37

APPENDIX A

EXPERIMENTAL ALGEBRA

CLASS RANDOMIZATION PLAN

APPENDIX A

ALGEBRA EXPERIMENT

CLASS RANDOMIZATION PLAN

The purpose of this memorandum is to assist the cooperating junior high schools in the assignment of students to four algebra classes according to a random process. It is essential to the future analysis of the data that one of the two plans outlined below be adhered to in organizing the algebra classes to be taught by the teacher who will attend the summer institute at Queens College.

The painstaking care involved in the class organization is required by the research design in order to equalize unknown differences between pupils assigned to experimental and control conditions. The attached abstract of the study indicates that the control group will be composed of two classes (Classes 1 & 2 in Figure 1) to be taught by the traditional method of group instruction. Also, the experimental group will be made up of two classes (Classes 3 and 4 in Figure 1) to be taught by the individualized method.

Plan A in the attached randomization procedure is designed for those schools in which ninth grade pupils do not exercise a choice of a foreign language or other curriculum elective. In other words, Plan A is designed for schools in which 120-160 pupils can be assigned to any one of four algebra classes without reztriction.

Plan B in the attached procedure is designed for those schools in which at least 60-75 pupils elect a foreign language or other special study and have to be considered as a block for class scheduling. In this instance we ask that the children be paired in alphabetical order for the forming of two random classes. One of the classes chosen under this plan will be assigned to the control condition and the other class to the experimental condition.

All four classes should be within one pupil of the same size. Those schedule makers in the cooperating schools who have difficulty with either Plan A or Plan B should telephone Professor H. E. Mitzel, Division of Teacher Education, TRafalgar 9-3600.

PLAN A

Instructions:

1. Alphabetize the 120-160 pupils scheduled for ninth year algebra by last name and enter their names in order below.

2. The number beside each student's name corresponds to his class (See Figure 1) for purposes of the investigation.

Pupil's Name	Class	Pupil's Name	Class
	2		1
	4		3
	1		2
	3		4
	2		2
	1		4
	3		1
	4		3
	3		4
	1		2
	2		3
	4		1
	3		4
	1		3
	2		1
	4		2
	2		1
	1		2
	3		4
	4		3
	2		4
	1		2
	3		1
	4		3
	2		4
	1		2
	3		1
	4		3

Pupil's Name	Class
________________	2
________________	1
________________	3
________________	4
________________	3
________________	2
________________	1
________________	4
________________	3
________________	4
________________	2
________________	1
________________	1
________________	4
________________	3
________________	2
________________	1
________________	3
________________	2
________________	4
________________	3
________________	1
________________	4
________________	2
________________	1
________________	4
________________	3
________________	2

Pupil's Name	Class
________________	2
________________	1
________________	4
________________	3
________________	2
________________	3
________________	1
________________	4
________________	1
________________	3
________________	2
________________	4
________________	1
________________	3
________________	4
________________	2
________________	1
________________	2
________________	3
________________	4
________________	4
________________	1
________________	3
________________	2
________________	1
________________	2
________________	3
________________	4

Pupil's Name	Class
	2
	3
	4
	1
	1
	4
	2
	3
	4
	1
	3
	2
	4
	1
	2
	3
	1
	4
	2
	3
	1
	2
	4
	3
	3
	2
	4
	1

Pupil's Name	Class
	2
	4
	1
	3
	3
	2
	4
	1
	4
	1
	2
	3
	2
	3
	1
	4
	2
	1
	4
	3
	4
	2
	3
	1
	3
	1
	2
	4

PLAN B

Instructions:

1. Alphabetize the 60-75 pupils who elect to take both ninth-year algebra and a special subject (such as a foreign language) into one block labelled B_1. Using the accompanying forms, two classes can be organized from this block of pupils, one of the classes should receive the experimental method of instruction, the other the control method. This same procedure can be followed for the 60-75 pupils in the second block (those not taking a foreign language).

BLOCK B_1

Pupil's Name	Class	Pupil's Name	Class
	3		3
	1		1
	3		3
	1		1
	3		1
	1		3
	1		1
	3		3
	1		1
	3		3
	3		1
	1		3
	1		3
	3		1
	3		1
	1		3

Pupil's Name	Class	Pupil's Name	Class
	3		3
	1		1
	1		3
	3		1
	3		3
	1		1
	3		1
	1		3
	1		3
	3		1
	3		1
	1		3
	1		1
	3		3
	3		1
	1		3
	1		1
	3		3
	3		1
	1		3

BLOCK B_2

Pupil's Name	Class	Pupil's Name	Class
	4		4
	2		2
	2		2
	4		4
	4		4
	2		2
	2		4
	4		2
	4		2
	2		4
	2		2
	4		4
	4		4
	2		2
	2		2
	4		4
	2		2
	4		4
	4		2
	2		4
	4		4
	2		2
	4		4
	2		2

Pupil's Name	Class	Pupil's Name	Class
	4		4
	2		2
	2		4
	4		2
	4		4
	2		2
	4		2
	2		4
	2		2
	4		4
	2		2
	4		4
	4		4
	2		2

APPENDIX B

ATTITUDE TOWARD MATHEMATICS SCALE

HOW DO YOU FEEL ABOUT MATHEMATICS?

This is not a test.

It is a chance for you to show how you feel about mathematics.

On the next three pages you will find 50 statements about mathematics.

You will probably agree with some of the statements. With other statements you will probably disagree. We are interested in your opinion about each of the statements.

You can tell us how you feel about each statement in the following way. First, read each statement carefully. Then, if you feel that the statement is true or if you agree with it, cross out "A" on the separate answer sheet. If you feel that the statement is false or if you disagree with it, cross out "D" on the separate answer sheet.

For example, suppose you were given this statement: I GET MY LOWEST MARKS IN MATHEMATICS. If you feel that this is true, that you get your lowest marks in mathematics, you should cross out "A" on the answer sheet. But if you feel that the statement is false, that you do not get your lowest marks in mathematics, you should cross out "D" on the answer sheet.

Be sure to mark a response for each statement even if you are not sure how you feel. Don't leave anything out.

There are 50 statements for you to read and respond to. Don't spend too much time on any particular statement. If you do, you may not finish.

Be sure to mark your response on the separate answer sheet. Just cross out or black out the letter that shows how you feel.

Remember: This is not a test. There are no right or wrong answers. Give your personal opinion.

1. It takes me a long time to catch on to a new topic in mathematics.
2. Very often in mathematics courses I cannot see a clear relationship between one topic and another.
3. I get a great deal of satisfaction out of solving a problem in mathematics.
4. I can't see how most of the mathematics I have learned thus far will really help me very much in later life.
5. In mathematics you have to be able to remember an awful lot of rules that don't make too much sense.
6. I find mathematics clear.
7. To do well in mathematics it's much more important to think clearly than to have a good memory.
8. Mathematics is such a hard subject that one student usually can't get very much help from another.
9. When I get an answer to a mathematical problem, I usually can't tell whether it's right or wrong until the teacher gives the correct answer.
10. Unless a mathematics teacher gives many quizzes, most students will soon fall behind.
11. In mathematics, ideas have a logical relationship to one another.
12. Mathematics is probably the most difficult subject in school.
13. Even before I begin a new topic in mathematics, I feel confident that I will be able to understand it.
14. Mathematics should be very appealing to a student with imagination.
15. I'm looking forward to studying some of the advanced mathematical topics I've heard about.
16. The trouble with mathematics is that it's too theoretical, and not practical enough.
17. I enjoy trying to solve mathematical problems and puzzles.
18. I think I have good mathematical ability.
19. The average student can't help being bored by mathematics.
20. I feel quite capable of going on to higher mathematics.
21. Mathematics helps us to find out more about the world we live in.
22. In mathematics you either know what you're doing or you don't, there's no in-between.
23. Unless a mathematics teacher gives a clear explanation of a topic, a student has difficulty.
24. I find mathematics useful in everyday life.
25. Mathematics is very interesting.

26. Mathematics courses are for the bright students, not those who are just average.
27. The only students who should be required to take mathematics are those who need it for a career like engineering or science.
28. Mathematics is an essential part of the background of a well-educated person.
29. Most of the students who get good marks in mathematics are "bookworms."
30. You don't have to have a special kind of abstract mind or an unusual mathematical talent to enjoy mathematics.
31. Mathematics frightens me.
32. In mathematics, more than in any other subject, what a student learns depends on how good the teacher is.
33. Mathematics is probably not the easiest school subject, but it isn't the hardest either.
34. Homework in mathematics is more difficult than homework in other subjects.
35. The most important thing in mathematics is a good memory.
36. Mathematicians are no more peculiar than doctors, lawyers, or people in other fields.
37. I would take mathematics even if I didn't have to.
38. Even when I understand a mathematical topic fairly well, I find it hard to explain it to someone else.
39. Mathematics is basically a very interesting subject, and there is no reason why a student has to find it boring or dull.
40. I get more nervous before a test in mathematics than a test in any other subject.
41. I find mathematics confusing.
42. Mathematics is highly practical as well as theoretical.
43. We always start a new topic in mathematics before I feel sure of the old one.
44. You don't need a special aptitude for mathematics to do well in the subject.
45. Students who are very good in mathematics are often not interested in other students.
46. An average student can understand mathematics.
47. In mathematics I have to memorize because I can't really understand.
48. In mathematics it isn't necessary for each student to study topics in the same order.
49. If you go about studying mathematics in a sensible way, you usually find it's not too difficult.
50. I like to study interesting applications of mathematics even if they are not a part of the assigned course work.

APPENDIX C

FINAL ACHIEVEMENT TEST *

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* We would like to express our appreciation to Professor Howard Fehr and D. C. Heath & Co. for permission to use some of the problems contained in the text, *Algebra I* by Fehr, Carnahan and Beberman.

QUEENS COLLEGE

Part A

Mathematics Individualized Learning Experiment

Gwladys Crosby, Principal Investigator
Herbert Fremont, Project Director, and Harold E. Mitzel, Consultant

Examination
in
First Year
ALGEBRA

TIME LIMIT: ONE HOUR

Directions to Pupils:

This test is planned to measure how much you have learned in your algebra classes during this past year. Some of the questions are hard, others are easy. YOU SHOULD NOT SPEND TOO MUCH TIME ON ANY ONE QUESTION. Answer the easier questions first, then return to the harder ones if you have time. You are not expected to be able to answer all of the questions, but do the best you can. Don't be alarmed if your neighbor has a different question booklet than yours. Try to answer every question, but avoid wild guessing. BE SURE TO PUT YOUR NAME ON THE ANSWER SHEET.

Spring, 1961

_____ M _____ U _____ P

Part A

Directions: BE SURE YOUR NAME IS ON YOUR ANSWER SHEET Check to see that your response number corresponds to the question number.

1. How many eighths are equivalent to 1/2 + 1/4?

1-1 2
1-2 4
1-3 6
1-4 8
1-5 12

2. What fraction, in lowest terms, is equivalent to .45?

2-1 4/50
2-2 1/4
2-3 9/20
2-4 5/30
2-5 1/5

3. $y^8 + y^2$ equals:

3-1 y^{10}
3-2 $10y$
3-3 y^6
3-4 y^{16}
3-5 $16y$

4. $r + s\ (3r)$ equals:

4-1 $4r + s$
4-2 $3r^2 + 3rs$
4-3 $r + 3rs$
4-4 $4r + 3s$
4-5 $r + 3s + r$

5. Solve the equation $EA = rs$ for the value of s when $r = 5$, $E = 10$, $A = 2$:

5-1 2
5-2 4
5-3 5
5-4 10
5-5 20

6. Subtract (+ 3) from (- 7):

6-1 10
6-2 -10
6-3 4
6-4 -4

7. In the formula $C = \frac{E}{R + r}$, if $C = 1$, $R = 12$, and $r = 8$, find E:

7-1 40
7-2 30
7-3 21
7-4 20
7-5 4

8. Energy is equal to half the product of its mass "m" and the square of its velocity "v". Written as a formula this statement is:

8-1 $E = 1/2\ mv^2$
8-2 $\frac{E}{2} = mv^2$
8-3 $E = 1/2\ (m + v)^2$
8-4 $\frac{E}{2} = m + v^2$
8-5 $E = m^2\ 2v^2$

9.

x	0	1	2	3	4	5
y	2	3	4	5	6	7

What is the formula that expresses the relation between x and y?

9-1 $y = x + 2$

9-2 $y = x + 1$

9-3 $y = 2x + 1$

9-4 $y = xy + 1$

9-5 $y = x + 7$

10. $x + 3$ is a factor of:

10-1 $x^2 - 2x - 15$

10-2 $x^2 + 9$

10-3 $x^2 - 2x + 15$

10-4 $x^2 - 2x + 9$

10-5 $x^2 - 6$

11. If x varies directly as y and $x = 20$ when $y = 4$, what does x equal when $y = 20$?

11-1 100

11-2 80

11-3 60

11-4 36

11-5 24

12. Simplify $-42 + (2a - 6a)$:

12-1 0

12-2 $-2a$

12-3 $-8a$

12-4 $+8a$

13. Simplify $(2a + 3b) + 2(a - 5b)$:

13-1 $4a - 7b$

13-2 $4a + 7b$

13-3 $2b$

13-4 $4a - 15b$

14. If a train runs d miles in 3 hours, how many miles will it run in h hours at the same rate?

14-1 $\frac{h}{3d}$

14-2 $\frac{3d}{h}$

14-3 $\frac{3h}{d}$

14-4 $\frac{d}{3h}$

14-5 $\frac{dh}{3}$

15. The volume of the sphere is equal to the product of the cube of the radius and four-thirds of π. Using r for the radius and V for the volume, the formula for the volume is:

15-1 $V = 4/3\ \pi r^3$

15-2 $V = 4/3\ \pi^3 r$

15-3 $V = 4/3\ (r^3 + \pi)$

15-4 $V = 4/3\ (\pi + r)^3$

15-5 $V = r^3 + 4/3\ \pi$

16. If the hypotenuse of a right triangle is 10 feet, and one leg is half the hypotenuse, how long is the other leg?

16-1 5

16-2 $5\sqrt{3}$

16-3 15

16-4 $2.5\sqrt{3}$

16-5 7

17. If the volume of a cube is 27 cubic inches what is the area of one of its faces in square inches?

17-1 3

17-2 9

17-3 12

17-4 3 1/3

17-5 9 2/3

18. Solve for x: $5x + 12 = -8$:

18-1 4/5

18-2 0

18-3 4

18-4 -4

19. If the perimeter of an equilateral triangle with side r, is equal to the perimeter of a square with side s, what does r equal in terms of s?

19-1 $r = \sqrt{s}$

19-2 $r = \frac{4s}{3}$

19-3 $r = \frac{9s}{16}$

19-4 $r = \frac{s}{12}$

19-5 $r = s - 1$

20. If a root of $x^2 + ax + b$ is $(x - 3)$, what is the value of b?

20-1 9

20-2 1

20-3 6

20-4 -9

20-5 -6

21. Add (+3) (-6) (+8) :

21-1 17

21-2 15

21-3 5

21-4 7

21-5 -7

22. Simplify (-6 + 14) - (-8):

22-1 16

22-2 10

22-3 8

22-4 4

23. 1/2 - 1/10 equals:

23-1 1/20

23-2 1/8

23-3 1/5

23-4 2/5

23-5 4/5

24. Find x: $4x - 6 = 6x + 12$

24-1 0
24-2 -6
24-3 4
24-4 -9
24-5 12

25. If $rs = 5/3$ and $rt = 7/3$, find $\frac{s}{t}$:

25-1 35/9
25-2 5/7
25-3 1/3
25-4 3
25-5 7/5

26. (.6) (.06) ÷ .2 equals:

26-1 1.8
26-2 .18
26-3 .018
26-4 .0018
26-5 .00018

27. 3/4 - 2/6 equals:

27-1 5/12
27-2 1/2
27-3 1/6
27-4 5/10
27-5 6/9

28. How much money must be invested at 3% simple interest for one year to earn $18.00?

28-1 $ 300.00
28-2 $ 600.00
28-3 $ 630.00
28-4 $ 900.00
28-5 $1,800.00

29. $y^5 \cdot y^4$ equals:

29-1 y^9
29-2 $9y$
29-3 y
29-4 y^{20}
29-5 $20y$

30. $y^2 + y^3$ equals:

30-1 $y^2 + y^3$
30-2 y^5
30-3 $2y^3$
30-4 $3y^2$
30-5 $2y^3$

31. $3(x + 2xy)$ equals:

31-1 $\frac{x + 2xy}{3}$
31-2 $3x + 6xy$
31-3 $9xy$
31-4 $5x + 3y$
31-5 $3x + 2xy$

32. In the formula $C = \frac{E}{R + r}$, what is the value of R when C = 3, E = 21 and r = 3?

32-1	20
32-2	9
32-3	7
32-4	6
32-5	4

33.

x	0	1	2	3	4
y	1	4	7	10	13

What is the formula that expresses the relationship between x and y?

33-1	$x = y-1$
33-2	$y = x+1$
33-3	$y = 3x+1$
33-4	$x = 3y-1$
33-5	$y = x+13$

34. Which of the following points is on the line $y = 3x - 5$?

34-1	0,5
34-2	1,2
34-3	-2,1
34-4	5,0
34-5	0,-5

35. If y varies directly as x, and y equals 12 when x equals 2, what does y equal when x equals 12?

35-1	100
35-2	82
35-3	72
35-4	62
35-5	26

36. Find x: $x + 6 = 2x + 1$:

36-1	6
36-2	-6
36-3	-5
36-4	2
36-5	5

37. Simplify: $\frac{2 + a}{a} - \frac{a + b}{b}$:

37-1	$\frac{2b + a^2}{ab}$
37-2	$\frac{2b - a^2}{a+b}$
37-3	$\frac{2b - a^2}{ab}$
37-4	$\frac{2b + a^2}{a+b}$
37-5	$\frac{2b + 2ab - a^2}{ab}$

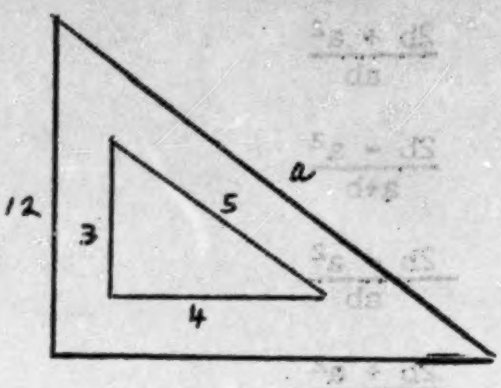
38. $\frac{ab + c}{9} = x$ Find a:

38-1	$\frac{9x - c}{b}$
38-2	$\frac{9x}{b + c}$
38-3	$9x - (b+c)$
38-4	$\frac{9x - b}{c}$
38-5	$\frac{x(9 - c)}{b}$

39. If r equals 2s, and $s \neq 0$, the fraction $\frac{3r + 4s}{9r - 4s}$ is equivalent to:

39-1 $-1/3$
39-2 $-1/3r$
39-3 $1/3$
39-4 $5/9$
39-5 $5/7$

40. The two triangles below are similar. What is the length of side a?



40-1 24
40-2 20
40-3 16
40-4 12
40-5 10

41. One side of a polygon is 2, and the corresponding side of a similar polygon is 3. Find the ratio of the area of the first polygon to the area of the second polygon:

41-1 3/2
41-2 9/4
41-3 2/3
41-4 4/9

42. A circle, whose center is at (7,2), passes through the point (2,14). Find the radius of the circle:

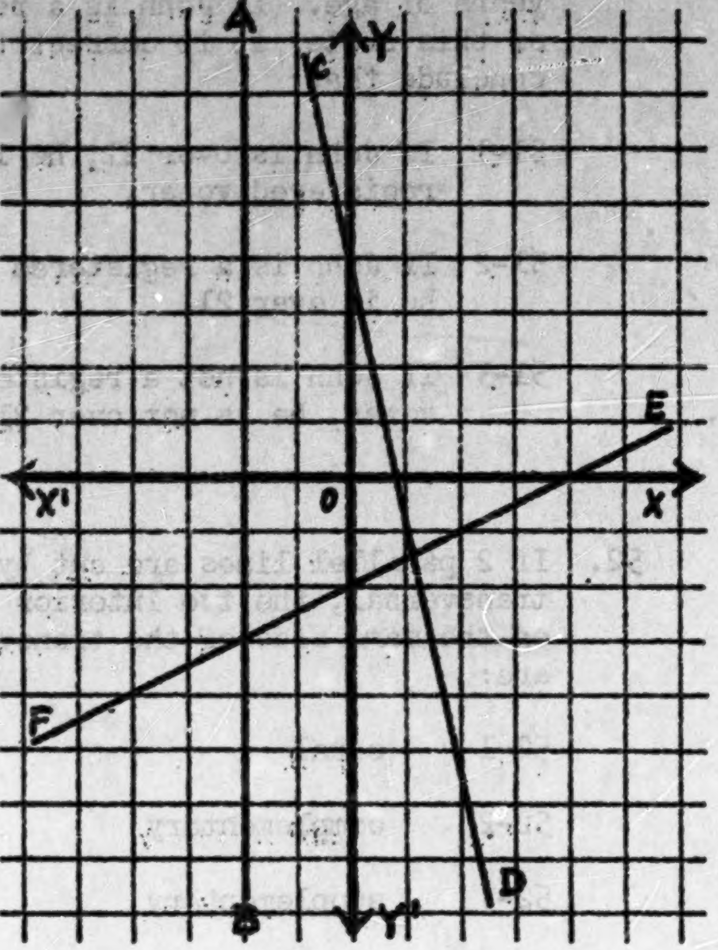
42-1 5
42-2 9
42-3 12
42-4 13
42-5 16

43. Find the number of degrees in an interior angle of a regular pentagon:

43-1 60°
43-2 90°
43-3 108°
43-4 120°
43-5 136°

44. In triangle ABC, angle C = 60°, and AB is greater than AC. Angle B is:

44-1 equal to 60°
44-2 less than 60°
44-3 greater than 60°



45. Using the above graph which of the following equations describes the line AB?

45-1 $y = 1/2\,x - 2$

45-2 $y = 2$

45-3 $y = -1/5\,x + 4$

45-4 $x = -2$

45-5 $y = -1/2x - 2$

46. Using the graph which of the following equations describes the line CD?

46-1 $y = 1/2\,x - 2$

46-2 $y = 2$

46-3 $y = -1/5\,x + 4$

46-4 $x = -2$

46-5 $y = -1/2\,x - 2$

47. Using the graph which of the following equations describes the line EF?

47-1 $y = 1/2\,x - 2$

47-2 $y = 2$

47-3 $y = -1/5\,x + 4$

47-4 [illegible] -2

47-5 $y = -1/2\,x - 2$

48. In the graph the point of intersection of lines EF and AB is:

48-1 (2, - 3)

48-2 (-2, - 3)

48-3 (-2, 3)

48-4 (2, 3)

49. Read the following four statements and answer the question below.

(A) An exterior angle of a triangle is equal to the sum of the non-adjacent interior angles.

(B) An angle formed by two secants is measured by 1/2 the difference of the intercepted arcs.

(C) An inscribed angle is measured by 1/2 the intercepted arc.

(D) A central angle is measured by its intercepted arc.

Which of the following represents the order in which the above statements should be arranged so that they form a logical sequence?

49-1 D, A, B, C

49-2 D, A, C, B

49-3 D, C, A, B

49-4 C, D, A, B

49-5 C, A, D, B

50. If line a is perpendicular to line b, and line c is parallel to a, then c and b are:

50-1 parallel

50-2 perpendicular

50-3 concurrent

51. All residents of this state who are registered voters are over 21 years of age. If John is a reside of this state, it is correct to conclude that:

51-1 If John is over 21, he is a registered voter.

51-2 If John is a registered vote he is over 21.

51-3 If John is not a registered voter, he is not over 21.

52. If 2 parallel lines are cut by a transversal, the two interior angl on the same side of the transversa are:

52-1 equal

52-2 complementary

52-3 supplementary

53. If perpendicular lines r and s intersect at O, and P is a point on r, the locus of points equidistant from r and s consists of:

53-1 a line through P.

53-2 two intersecting lines.

53-3 two parallel lines.

54. What is the name of the graph of the equation $ax^2 + by^2 = c$ in which a, b and c are positive and $a \neq b$?

54-1 parabola

54-2 ellipse

54-3 circle

54-4 hyperbola

55. What is the discriminant of the equation $2x^2 - 5x - 2 = 0$?

55-1 3

55-2 $\sqrt{41}$

55-3 9

55-4 41

56. The value of $2^3 \times 2^{-1} + 2^0$ is:

56-1 4

56-2 5

56-3 9

56-4 12

57. The sum of $3\sqrt{-4}$ and $\sqrt{-9}$, in terms of i, is:

57-1 3i

57-2 6i

57-3 9i

57-4 18i

58. The first term of an arithmetic progression is -4 and the twelfth term is 10. The sum of the 12 terms is:

58-1 96

58-2 84

58-3 36

58-4 24

59. What is the equation of the straight line which passes through the origin and has a slope of 3?

59-1 $y = 3x$

59-2 $y = 1/3x$

59-3 $y = 1/3x + 2$

59-4 $y = 3x + 2$

60. The value of $64^{3/2}$ is:

60-1 -512

60-2 1/16

60-3 6/512

61. The roots of the equation $3x^2 - 5x - 2 = 0$ are:

61-1 real, rational, equal

61-2 real, rational, unequal

61-3 real and irrational

61-4 imaginary

62. Solve for x: $\sqrt{x - 3} - 2 = 0$

62-1 0

62-2 1

62-3 5

62-4 7

63. Find a value of x, and a value of y that satisfy the equations:

$y = x^2$
$y = 2x - 1$

63-1 (0,0)
63-2 (0,1)
63-3 (1,0)
63-4 (1,1)

64. Which of the following sentences has the same truth set as the sentence "$x = 5$"?

64-1 $x < 5$ or $x = 5$
64-2 $x \ngtr 5$
64-3 $x \nless 5$
64-4 $x \neq 5$

65. Solve the following equation:
$|x| + (-3) = |-2| + 5$

65-1 -10
65-2 - 6
65-3 6
65-4 10

66. The set $\{-5, 0, 3/4, 5, 7\}$ is closed under the operation of:

66-1 addition
66-2 taking the absolute value of each element
66-3 taking the opposite of each element

67. Given the set $V = \{1, -2, -3, 4\}$ find the set K of all positive numbers obtained as products of pairs of elements of V.

67-1 $K = \{4, 6\}$
67-2 $K = \{2, 3, 4, 6\}$
67-3 $K = \{2, 3, 4, 6, 8, 12\}$

68. What is the average of the numbers:

$\frac{x+3}{x}, \frac{x-3}{x}, \frac{x+k}{x}, \frac{x-k}{x}$?

Note: $x \neq 0$

68-1 0
68-2 1
68-3 4
68-4 $\frac{x+6+k}{x}$

69. Which of the following is a polynomial over the integers:

69-1 $3t + 1$
69-2 $t + 1/3$
69-3 $rq - \sqrt{2}$
69-4 $\frac{x+y}{2}$

70. The reflexive axiom of equality states that for every number x:

70-1 $x = 2, \ a = b, \ x = b$
70-2 $x = x$
70-3 $x = a = x + b, \ x = b$

71. From the 2 equations: a
$(b+c)x = x(b+c)$ and $x(b+c) = xb + xc$,
we may infer that $(b+c)x = xb + xc$
by the:

71-1 associative law

71-2 distributive law

71-3 commutative law

72. The statement "$11 + 4 = 4 + 11$"
may be inferred from the:

72-1 associative law

72-2 distributive law

72-3 commutative law

73. A rule that permits us to make certain rearrangements of parentheses in sums of products without changing the number which is represented is called the:

73-1 associative law

73-2 distributive law

73-3 commutative law

74. We have proved the statement
$x = -x \longleftrightarrow x = 0$, which states that the number 0 is the one and only number that is its own:

74-1 multiplicative inverse

74-2 additive inverse

74-3 multiplicative identity

74-4 additive identity

Part B

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PRINT	Last Name	First	School	Class

Spring, 1961

M	U	P

Mathematics Individualized Learning Experiment

Part B

Directions: Read each problem carefully and put your answer on the question booklet in the space provided below and to the right of the problem. Only one answer is called for on each problem. The separate scratch paper is to be returned with the booklet.

1. If y is Mary's age now, how old was she 5 years ago?

Ans. 1 ____________

2. Bob bought a radio for $25.00. He is going to pay for it at the rate of x dollars per week. How long will it take him to pay for it?

Ans. 2 ____________

3. Two gears have 8 teeth and 28 teeth respectively. The larger gear makes 8 turns each second.. How many turns each second is made by the smaller gear?

Ans. 3 ____________

4. The perimeter of a rectangle is 36". If the length is 2 inches greater than the width, find the length.

Ans. 4 ____________

5. If the sum of 2 numbers is 10 and their difference is 4, find the larger number.

Ans. 5 ____________

6. A lot of 3 cent and 4 cent stamps is worth $8.25. There are 250 stamps in all. How many of each kind are there in the lot?

Ans. 6 ____________

7. Mr. East drove 1198 miles in 3 days. At the same rate, how far would he drive in 7 days?

Ans. 7 ____________

8. A man receives $106 a year from two investments. The total amount of money invested is $2300. The two investments earn interest at the rates of 4% and 5% respectively. How much money is invested at 5%?

Ans. 8 ____________

9. The difference between 10 and 6 is 10 - 6.

The difference between 7 and 2 is 7 - 2.

Write the difference between x and y.

Ans. 9 ____________

10. 7% of a number = $\frac{7}{100}$ of that number.

5 1/2% of a number = $\frac{5\ 1/2}{100}$ of that number.

x% of a number = —— of that number.

Ans. 10 ____________

11. The sum of three consecutive odd integers is 51. Find the largest integer.

Ans. 11 ____________

12. In a light house, from the observation post, which is 120 feet above sea level, the keeper sees a boat at an angle of depression of 18°. How far is the boat from the base of the lighthouse?

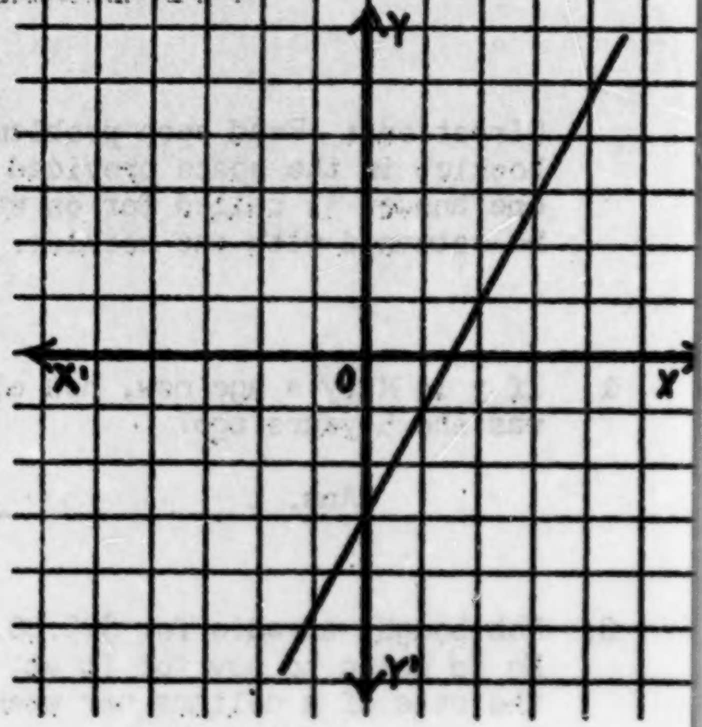
Ans. 12 ____________

13. By what amount must the radius of a circle be increased in order to increase the circumference 11 inches?

Ans. 13 ____________

14. The sum of the digits of a two-digit number is 6. If the digits are interchanged, the new number is 18 less than the given number. Find the given number?

Ans. 14 ____________



15. In the above graph what is the x intercept?

Ans. 15 ____________

16. In the above graph what is the y intercept?

Ans. 16 ____________

17. If sugar costs 10¢ a pound you can get 6 pounds for 60¢. If sugar costs 15¢ a pound you can get 4 pounds for 60¢. If sugar costs x¢ a pound, how many pounds can you get for 60¢?

Ans. 17 ____________

18. If $5 - 3 = 2$, then $5 = 2 + 3$.

If $20 - 6 = 14$, then $20 = 14 + 6$.

If $a - b = c$, then $a = ?$

Ans. 18 ____________

19. A dealer has some cashew nuts that sell at $1.20 a pound, and almonds that sell at $1.50 a pound. How many pounds of almonds should be put into a mixture of 200 pounds to sell at $1.32 a pound?

Ans. 19 ____________

20. The dimensions of a rectangular block are in the ratio of 2:3:4. Its total surface area is 208 square inches. Find the dimension of the smallest edge.

Ans. 20 ____________

21. Looking out to sea from the top of a cliff, "h" feet high, you can see a distance "d" miles, such that $2d^2 = 3h$. If the cliff is 300 feet high, how far can you see?

Ans. 21 ____________

22. A photographic negative is 3" by 4". An enlargement was made whose width is 7 1/2". What is its length?

Ans. 22 ____________

23. The sum of a number and its reciprocal is equal to twice the number. Find this number.

Ans. 23 ____________

24. Express the following statement in the form of an equatioh, without simplifying:
The difference between the squares of two consecutive whole numbers is 39.

Ans. 24 ____________

25. The angle of elevation of an airplane is 58°. The altitude of the plane is 8500 feet. What is the horizontal distance from the observer to a point directly below the plane?

Ans. 25 ____________

26. What is the two digit number which has x for its tens digit and y for its units digit? The number is 18 greater than it would be if the digits were reversed, and the sum of the digits is 8. Find the number.

Ans. 26 ____________

27. The distance between city A and city B is 375 miles. A 34 mile per hour train leaves city A for city B. At the same time, a 41 mile per hour train leaves city B for city A. How far from city A will they meet?

Ans. 27 ____________

28. Two boys each sitting at an end of a 10-foot teeterboard are in balance when the fulcrum is 4 1/2 feet from the heavier boy. The heavier boy weighs 22 pounds more than the lighter boy. Find the lighter boy's weight.

Ans. 28 ____________

29. Twenty pounds of 72¢ a pound coffee and a certain number of pounds of 85¢ a pound coffee are combined to make a mixture worth 80¢ a pound. What is the total value of the mixture?

Ans. 29 ____________

30. Find the number which is halfway between a - b and b - a.

Ans. 30 ____________

31. If x represents Bob's age now, how old will he be six years from now?

Ans. 31 ____________

32. Rickey has 3 dollars more than twice as much money as Fabian. If Fabian has "n" dollars, how much does Rickey have?

Ans. 32 ____________

33. If you get an allowance of "d" dollars a week, how much would that average per day?

Ans. 33 ____________

34. The diagonal of a rectangular lot is 50 feet, and the length of the lot is 40 feet. What is the area of the lot?

Ans. 34 ____________

35. Separate 47 into two parts which are in the ratio of 4/7.

Ans. 35 ____________

36. A 12 foot ladder when placed agains the side of a house makes an angle of 34° with the ground. How far f the house is the foot of the ladder

Ans. 36 ____________

37. Am alloy contains 60% copper and 4 lead. How many pounds of copper should be added to 70 pounds of thi alloy to produce a new alloy contai ing 70% copper?

Ans. 37 ____________

38. Ann works in a production room of the Acme Mailing Service inserting letters in envelopes. If Ann and her friend insert 4500 letters in 7 hours, how many letters could 12 girls working at the same rate insert in the same time?

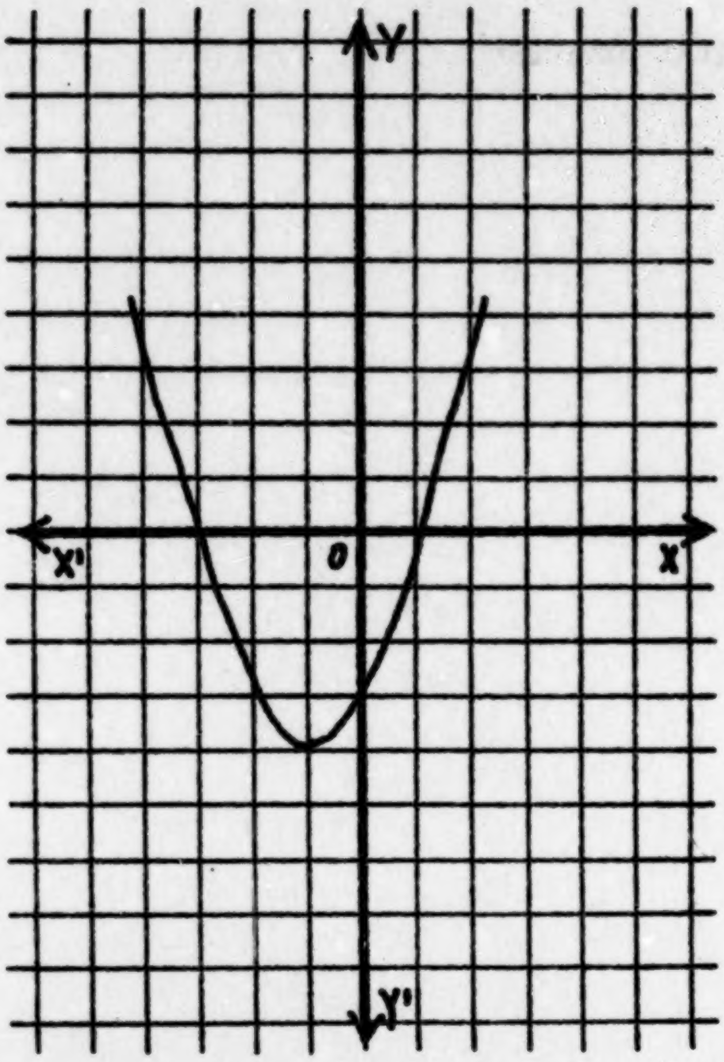
Ans. 38 ____________

39. At its last football game Longview High School sold $130 worth of tickets. Adults were charged 55¢ and students were charged 35¢. If 320 tickets were sold, how many of them were adult tickets?

Ans. 39 ____________

40. A can do a job in 4 hours. He works on this job for an hour and a half. B, who can do the same job in 3 hours, replaces A. How long must B work to complete the job?

Ans. 40 ____________



41. In the above graph what is the positive value which makes y equal zero?

Ans. 41 ____________

42. From the graph above give the coordinates of the minimum point.

Ans. 42 ____________

APPENDIX D

LIST OF PERCEPTUAL DEVICES*

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* Lack of space prevents full explanation of all devices; this information is available at: MILE, Queens College, Flushing 67, New York

LIST OF PERCEPTUAL DEVICES

1. *The Hill - Formulas, Linear and Quadratic Equations
2. *Gear Board - Multiplication of Directed Numbers
3. Numbers You Can See - Basic Numerical Operations
4. Keeping Equals Equal - Linear Equations
5. *The Factor Finder - factoring polynomials
6. Introduction to Cuisenaire Rods - Factoring
7. Circle with Variable Radius - Trigonometry
8. Indirect Measurement - Pythagorean Theorem
9. An Unusual Triangle - Similar Triangles - Space Measurement
10. From Squares to Rectangles - Factoring the Difference of Two Squares
11. Operations You Can See - Basic Algebraic Operations
12. Quadratics
13. Introduction to Graphing - Graphing
14. The Shape of Numbers - Symbolism and Basic Operations
15. Operations with Signed Numbers
16. *Scrabble Solved Systems
17. *The Lever - Variation
18. Picture Puzzle - Plotting Points, Review of Operations with Signed Numbers
19. *The Frame - Multiplication of Binomials
20. Cuisenaire Rods for Linear Equations
21. Solving Systems With Rods
22. Cuisenaire Rods for Factoring

* See Appendix F

23. Geometry for Square Roots - Geometric Construction Square Roots
24. The Swinging Rods--Proportions of Lines
25. The Clinometer - Indirect Measurement
26. Cuisenaire Rods - Fundamental Operations
27. Understanding Polynomials - Multiplication of Polynomials
28. Changeable Right Triangles - Trigonometric Functions
29. Use of Cuisenaire Rods to Understand Ratio
30. Fractions - Basic Operations
31. The Doll and the Airplane - Numerical Trigonometry
32. *The Balance - Equation
33. The Pulley - The Meaning of Function
34. The Water Spout - Linear Function
35. Operations with Expontial Numbers
36. Dividing Numbers and Letters
37. *Arrows Point the Way - Operations With Signed Numbers
38. Natural Laws to Shorthand - Formulas
39. Numbers from Outer Space
40. Binary System
41. Unthings - Negative numbers
42. The Backwards Book - Division of Polynomials
43. Directed Numbers
44. From Arithmetic to a Quadratic
45. *Power Finder
46. Exponents
47. Identities
48. Uses of Square Roots
49. Teach-Yourself-Cards -- Motion
50. The Room
51. Cross-Number Puzzle

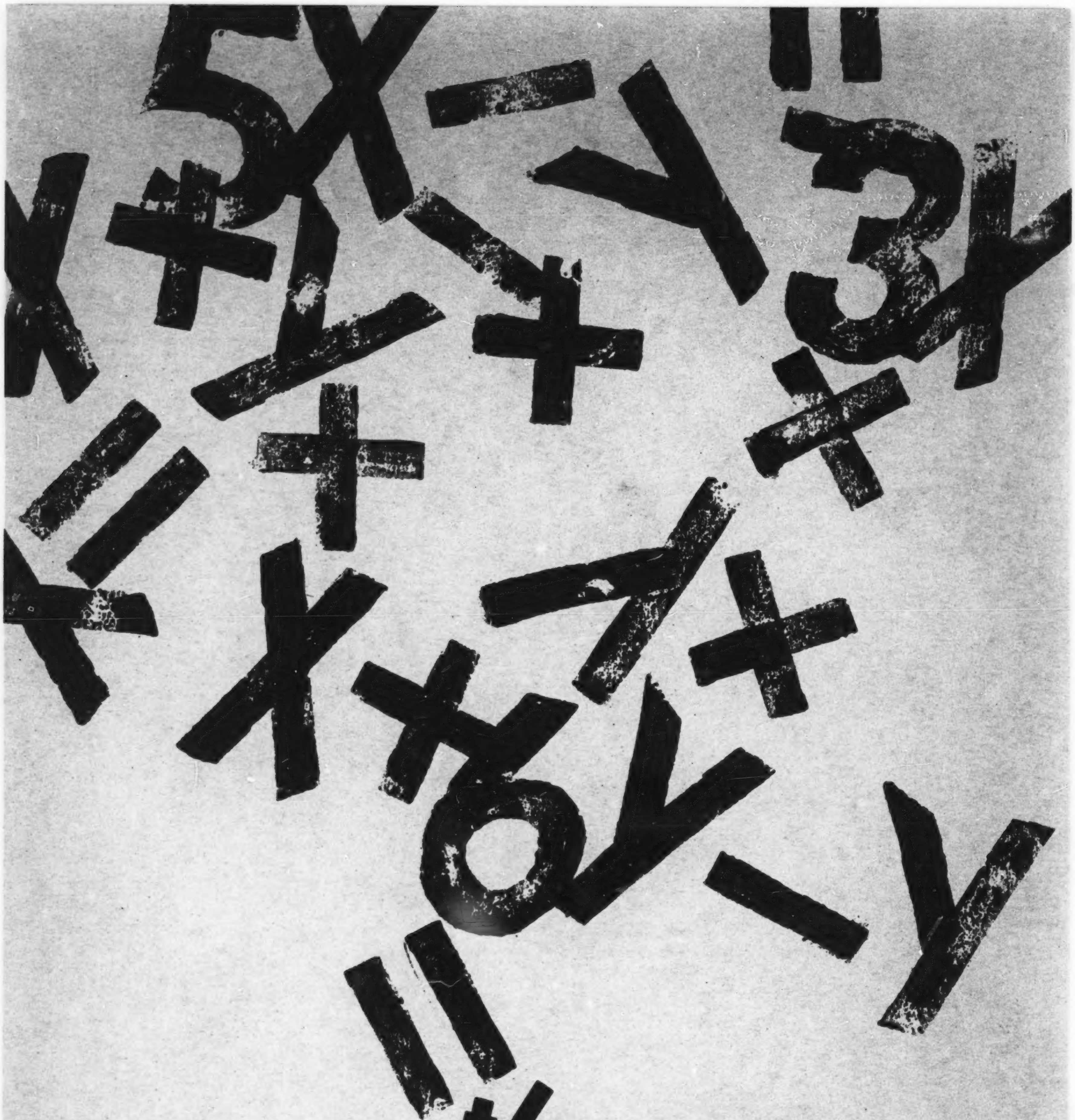
In addition to the perceptual devices listed above, all units were made available to the participating schools in ditto and a limited number on tape and in programmed form.

Use was made of Temac, Tutortexts, T.M.I. Programs and Films supplied by the New York State Education Department, Albany, New York

APPENDIX E

THE MATH LAB

Because of space limitations, only one experiment is included herein; the whole manual is available at: MILE, Department of Education, Queens College, Flushing 67, N. Y.


5X
=
3
Y
X
+
X
=
+
Y
X
+
+
Y
X
+
Y
+
6
Y
=
y

the math lab

VELOPED AT THE MILE INSTITUTE QUEENS COLLEGE , FLUSHING, N.Y.

ported in part under provisions of Title VII National Defense Education Act 1958

To the Student:

This booklet is in the form of a laboratory manual containing a set of experiments. The equipment you will need, and the steps (procedure) you must follow are part of each experiment. We hope that you will discover certain mathematical ideas for yourself through performing these experiments.

A manual of your own will allow you to perform the experiment in the time you need. You may work as slowly or as quickly as you feel you should in order to finish the work successfully.

****

To the Teacher:

This booklet was written directly to the student in words which we believe a ninth grader will have no difficulty understanding.

We hope that individual manuals will enable each student to work at his own pace. The faster student will not be kept back by the slower ones; in turn, the slower one will not be forced to maintain too fast a pace for complete mastery. The student who sees the application of algebra in our universe should find the course meaningful.

Each experiment follows accepted scientific procedure. In addition, each student is asked to summarize what he has learned so that he will mentally review what he has done.

Wherever possible, the answer is included.

THE MATH LAB

INTRODUCTION

Basic Idea of Algebra

What is algebra? Perhaps you had some algebra last year and you know that it is a kind of shorthand, a brief way of expressing laws. It is a new language to help you understand the laws of nature.

The first experiment, Easy As Pi, has been designed to help you discover a law of nature. After you have found it, you will be able to use it as a fact without having to prove it again. You will discover other laws or rules which you will be able to use later. Mathematicians have worked out many theorems and rules which you can prove if you wish. This manual is only a beginning.

You will notice that Experiment 5 is missing. That is because you will probably want to make up an experiment yourself by the time you get to No. 5. Since mathematics is something that is everywhere, you should begin to look around for experiments that you can do yourself. For example, you know some laws such as: $A = lw$ and $I = prt$ and $P = 2(l-w)$. Can you set up experiment 5 so as to show how the very first person discovered one of these rules?

Experiment 1 - Easy as Pi

Purpose: To see if you can discover the relationship between the circumference and the diameter of a circle.

Collect these items:

Tape measure
String
Ruler with centimeter markings
Six tin cans with circular tops

Procedure:

Answers (a) and (b) follow the experiment.

1. Measure the circumference (c) of one of the cans with a tape measure and record it on the first line under C. Record
2. Measure the diameter (d) of the circle. Record it.
3. Divide the circumference by the diameter and carry out to a three place decimal. Record that in the appropriate column.
4. Repeat the same procedure for two of the other cans.

5.

Table

Object	C	d	$\frac{C}{d}$
(1)			
(2)			
(3)			
(4)			
(5)			
(6)			

6. Repeat the same procedure three more times with other cans but use a string as a tape measure. To measure the string use the centimeter rule.

7. What do you notice about all of the results in the last column? ___(a)___ Can you write a statement about the ratio of the circumference to the diamete $\left(\frac{C}{d}\right)$ for any circle no matter how large or how small? ____________________(b)

8. Take the average of the values in the last column $\left(\frac{C}{d}\right)$ because this procedure tends to cancel the errors in measurement of each individual trial, and place the average here. __________________

We call the ratio of the circumference to the diameter of a circle pi (π) and give its approximate value as 3.14.

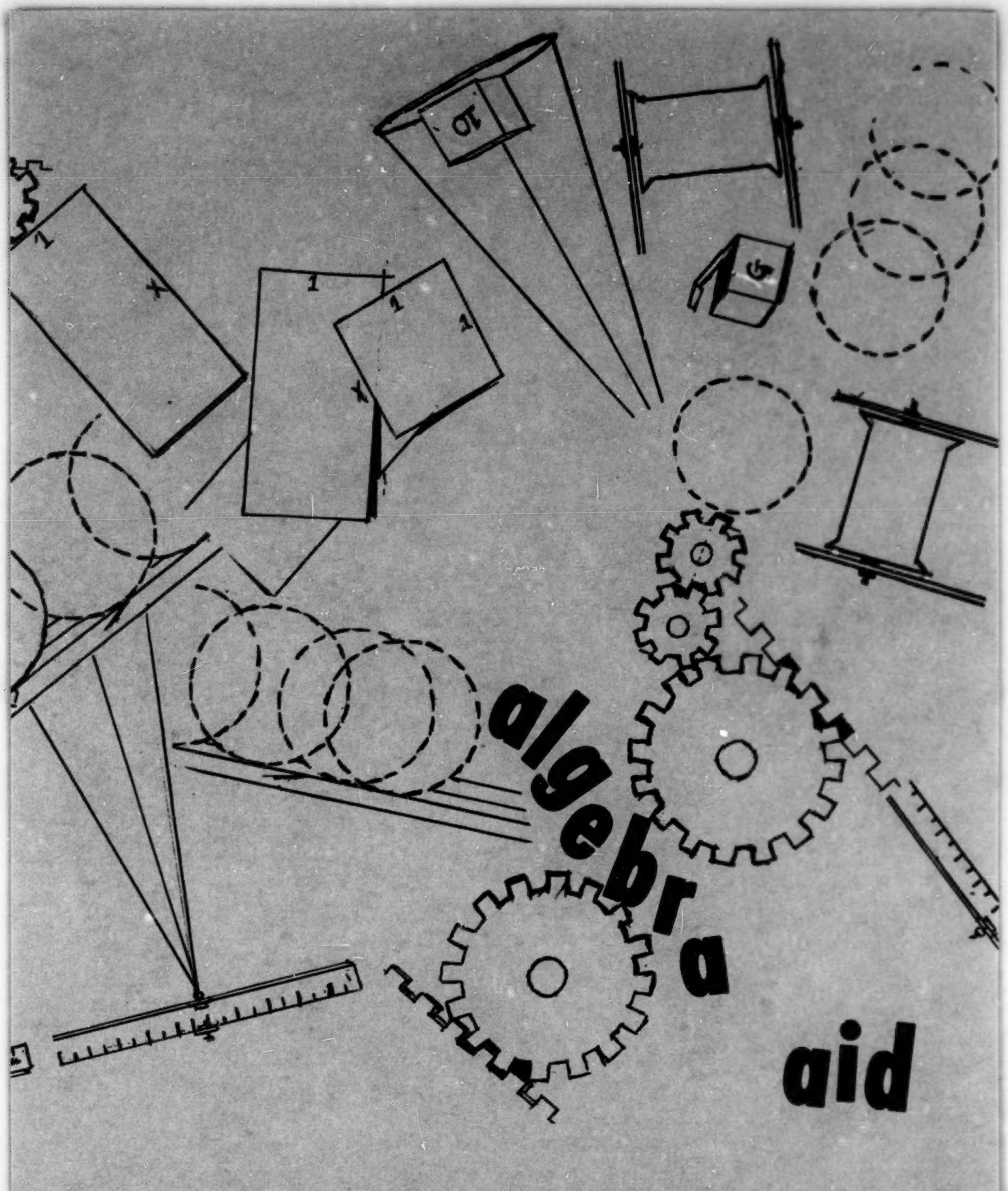
9. What would be the circumference of the object which had a doubled diameter? ______________________________

What have you learned?

Answers

a. They are almost equal.

b. The ratio of the circumference to the diameter is a constant.



algebra aid

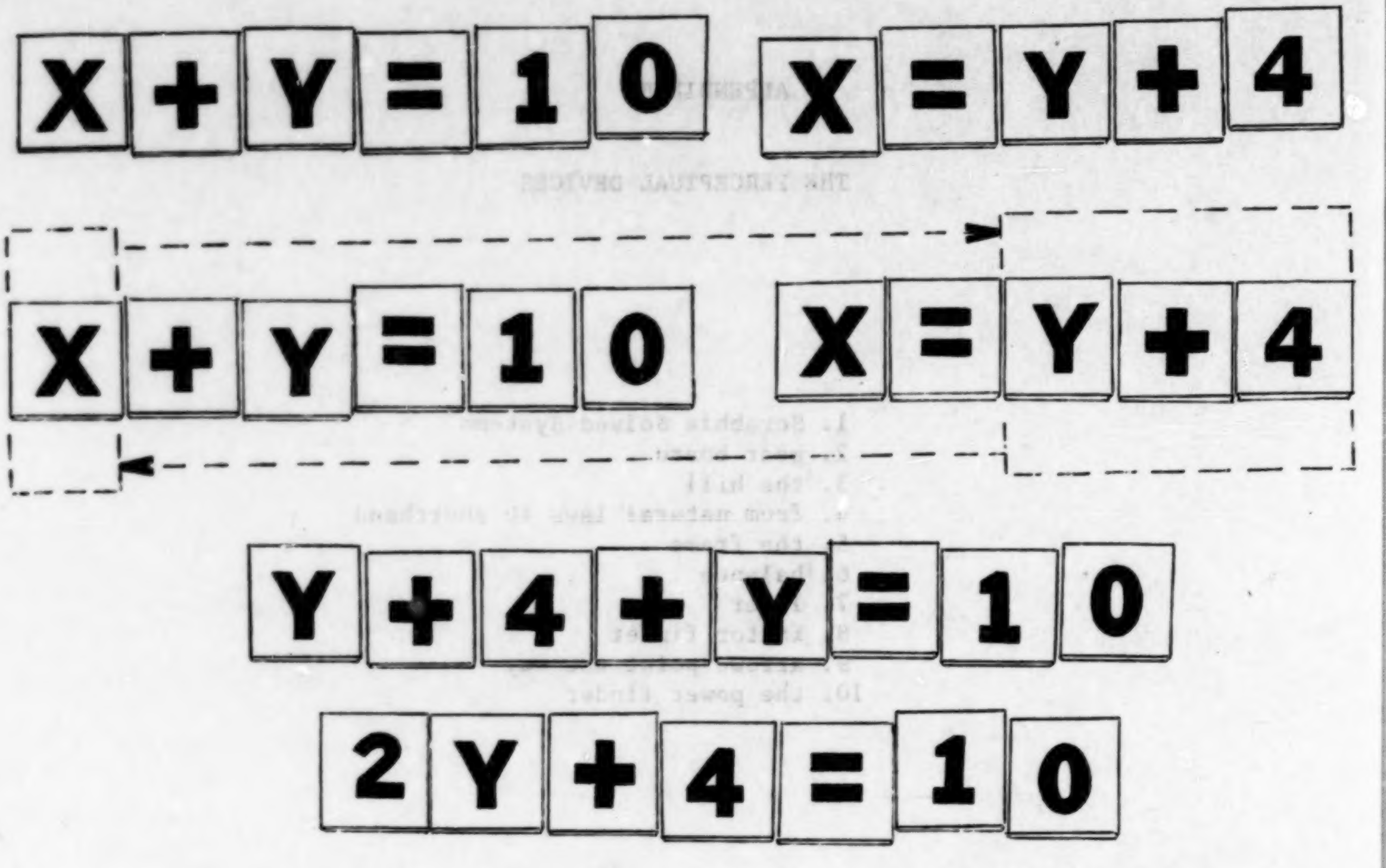
MILE INSTITUTE QUEENS COLLEGE, FLUSHING, N.Y.

rted in part under provisions of Title VII National Defense Education Act 1958

APPENDIX F

THE PERCEPTUAL DEVICES

1. Scrabble Solved Systems
2. gear board
3. the hill
4. from natural laws to shorthand
5. the frame
6. balance
7. lever
8. factor finder
9. arrows point the way
10. the power finder


X + Y = 10
X = Y + 4
X + Y = 10
X = Y + 4
Y + 4 + Y = 10
2Y + 4 = 10
Y = 3
X + Y = 10
X = Y + 4
X + 3 = 10
X = 3 + 4
X = 7

SCRABBLE SOLVED SYSTEM

APPENDIX F

No. 1 Scrabble Solved System

1) If I told you I was thinking of two numbers could you tell me what they are?

2) Suppose I told you that these two numbers add up to 10, could you guess what they might be? Write your guess. Can you think of any other pair of numbers that add up to 10?

There are quite a few, aren't there? Let's arrange them in a table similar to this
Copy and complete.

10					
0					

Did you include any negative numbers? Could you? Extend the horizontal line in your table and enter a few more pairs of numbers so as to include negative numbers.

Well, there are quite a few numbers that add up to 10. But you couldn't tell which two I was thinking of, could you?

3) I'll give you another hint. First I told you the two numbers add up to 10. Now, suppose I said that one of these two numbers is 4 more than the other. Now look at your table - do you have a pair of numbers that fit both these conditions? Draw a circle around them.

This took a long time -- it can be done very quickly with just a little knowledge of algebra. Want to see?

4) Suppose we wanted to represent any pair of these numbers that add up to 10. You know enough about variables. (Remember, variables are merely place holders until you discover the number to replace them). Use any letters you wish and enter them in your table, one at the beginning of each row. Then your first variable could hold the place for any number you have in the top row of your table, and the second variable could hold the place for any number you have in the bottom row.

5) Now let's translate the conditions into algebraic language.

a) Two numbers add up to 10. Write equations using your two variables.
b) One number is 4 more than the other. Write this as an equation. For one numeber use one variable and state in equation form that it equals four more than the other variable.

6) What do you have now? It should be 2 equations each containing 2 variables.

$x + y = 10$ $x = y + 4$ or $y = x + 4$ or $x - y = 4$

See if you can set these equations up with the Scrabble game.

7) Do you know how to start solving such a pair of equations?

Go back to what you do know! What kind of equations can you solve? And how are these equations different from the ones you can solve?

8) Look at the equation on the scrabble racks. Be careful -- they're easily moved.

One says the sum of 2 numbers is 10.

The second says one of these numbers is 4 more than the other.

Could the second equation -- or a part of it -- help you to change the first equation into a form that you could solve? (I know you can solve an equation with only one kind of variable.)

Look hard -- Do you see anything in the second equation that is related to something in the first equation? (What does it say x - or whatever variable you used - is equal to?)

Now -- could you use that equality in the first equation? Try it - take what the first variable in the second equation equals (the right member of the equation) and put it in the plcae of the variable in the first equation, to which it refers.

9) You should have an equation, now, that you can solve -- that is, an equation with only one kind of unknown. Do you? Then solve it.

If you worked correctly you now have a value for 1 variable. What could you do with this value that would help you to find the value of the other variable?

Find a scrabble square equal to the value you found.

Fit this number into the place of the variable in either equation.

Solve this new equation.

Finished? Did you find from your equation that one variable has the value of 3 and the other has the value of 7?

10) What did you actually do each time when you removed a variable and put something equal to it in its place?

What does Casey Stengel send out to center field when Mickey Mantle isn't doing well?

Who takes my place when I'm absent?

Well - we solved a pair of equations by SUBSTITUTION.

This is an axiom, by the way. See if you can state the substitution axiom.

11) Let's go back to our 2 equations on the scrabble racks

$x + y = 10$

$x = y + 4$

To check: Substitute in each equation 7 for x, 3 for y.

Do you have 2 true statements?

Then your answer is correct.

12) Of course, every system of equations isn't ready-made for you to solve by substitution. But, by using your axioms, you can quickly make them so. Look at this:

$x + y = 10$ The difference between 2 numbers is 4.

$x - y = 4$ Is that the same as: One number is 4 more than the other?

If you wish to solve by substitution what could we do with the second equation, $x - y = 4$, to put it into the form $x = y + 4$?

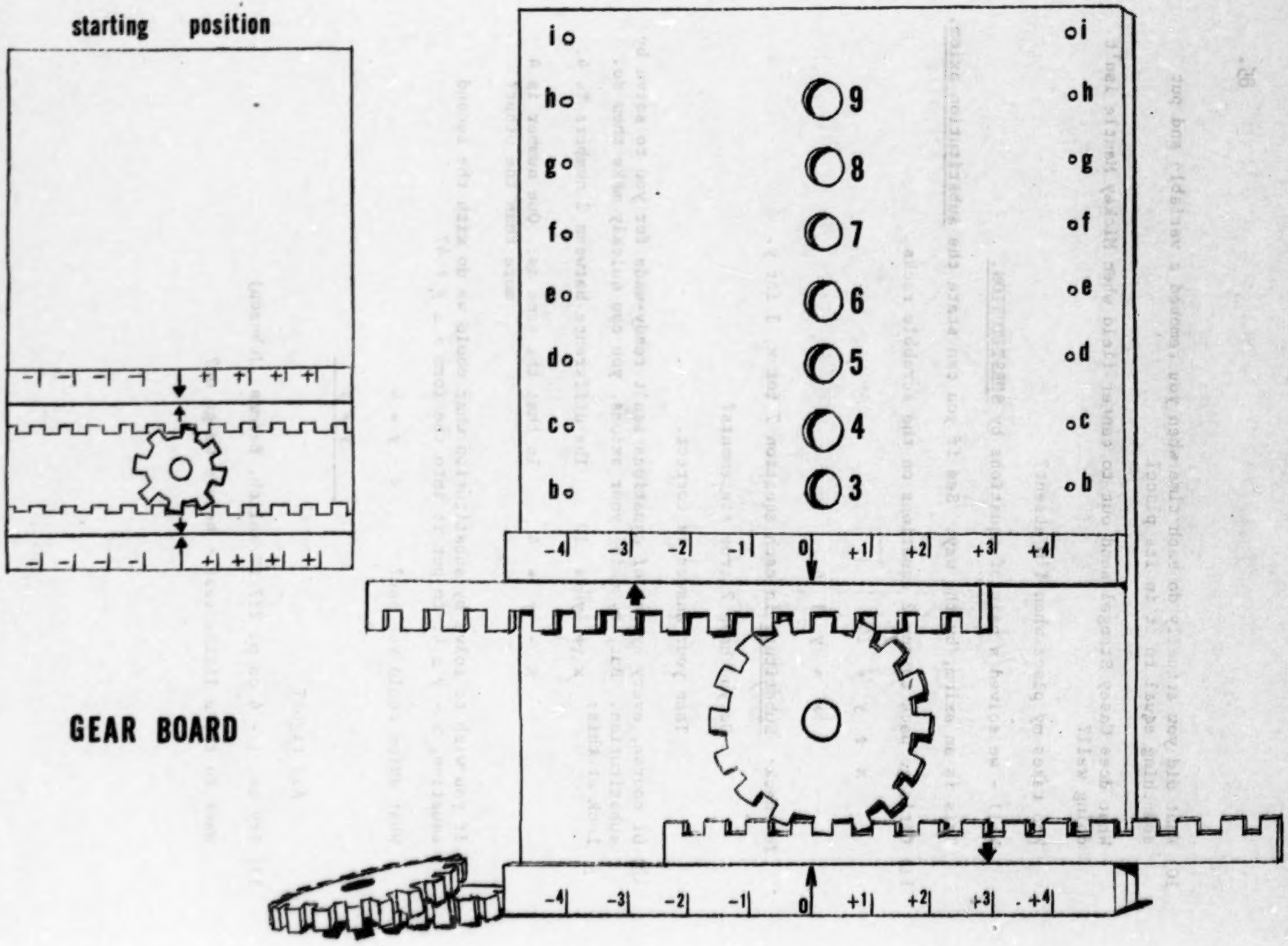
What axiom could you use? $x - y = 4$

$\underline{\quad y = 7 \quad}$

All right?

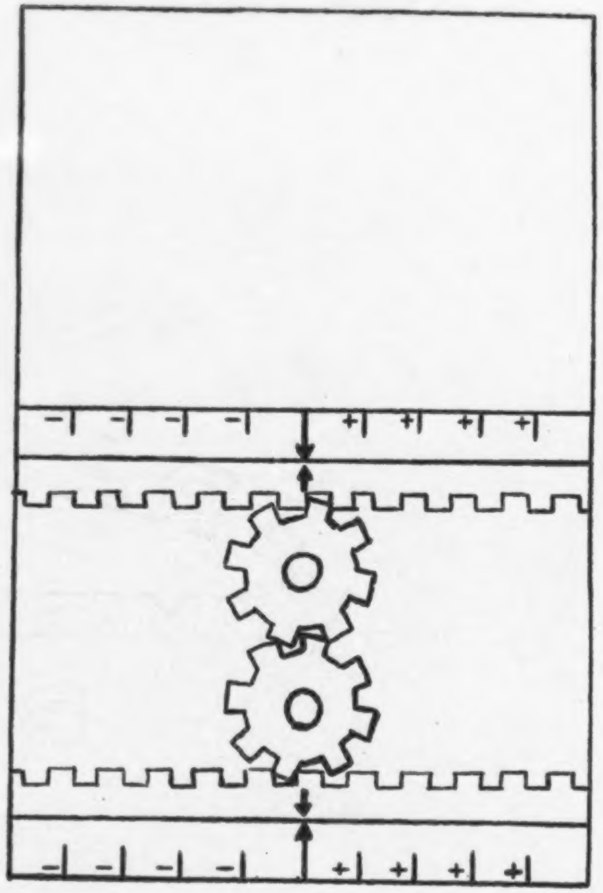
13) Try ex. 1 - 4 on p. 237 (Froelich, Berman, Johnson)

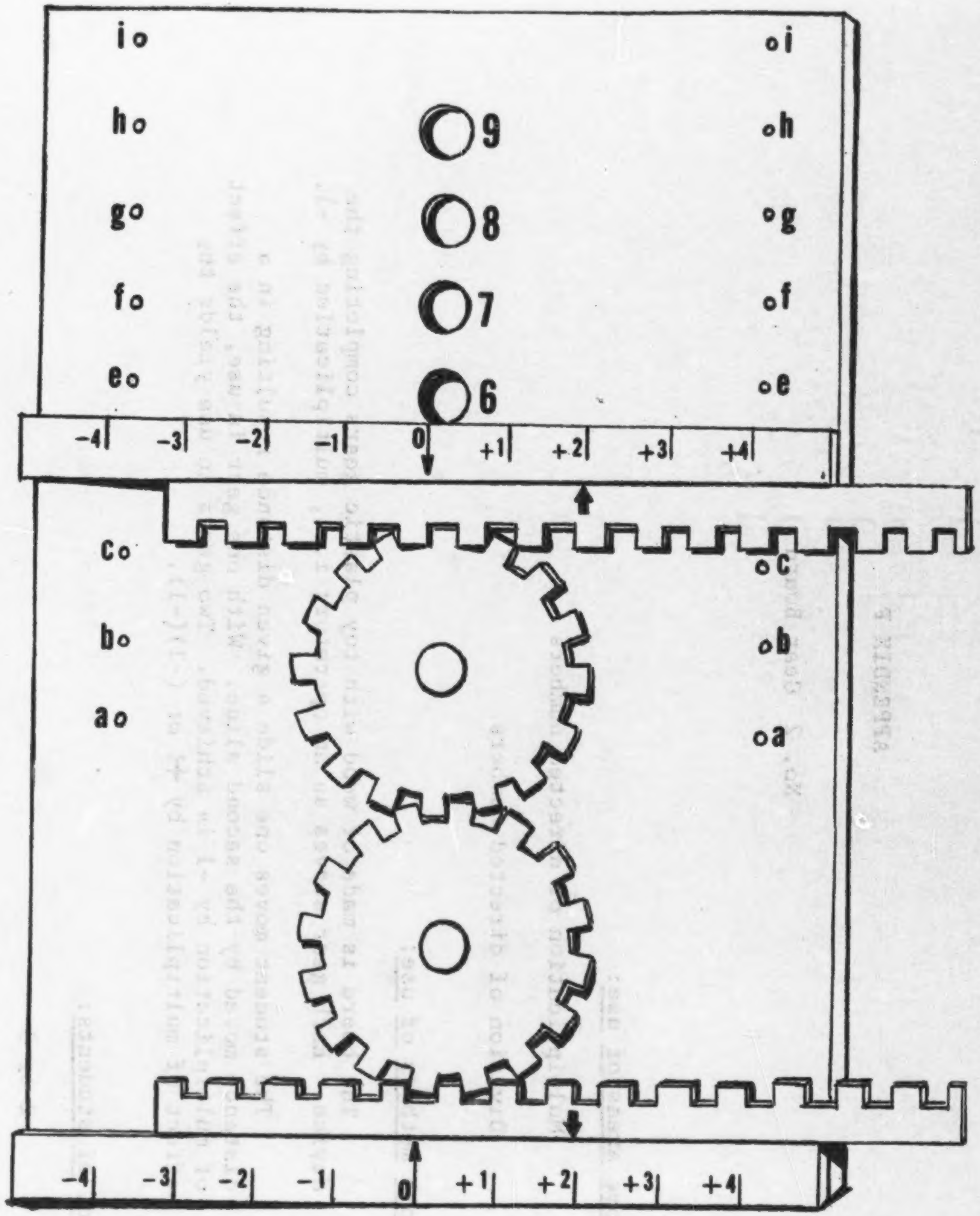
Want to try a little test - before we go on?


starting position
i h g f e d c b
9 8 7 6 5 4 3
-4 -3 -2 -1 0 +1 +2 +3 +4

GEAR BOARD

starting position




i
h
g
f
e
c
b
a
9
8
7
6
−4
−3
−2
−1
0
+1
+2
+3
+4

GEAR BOARD

APPENDIX F

No. 2 Gear Board

Some areas of use:

Multiplication of directed numbers

Division of directed numbers

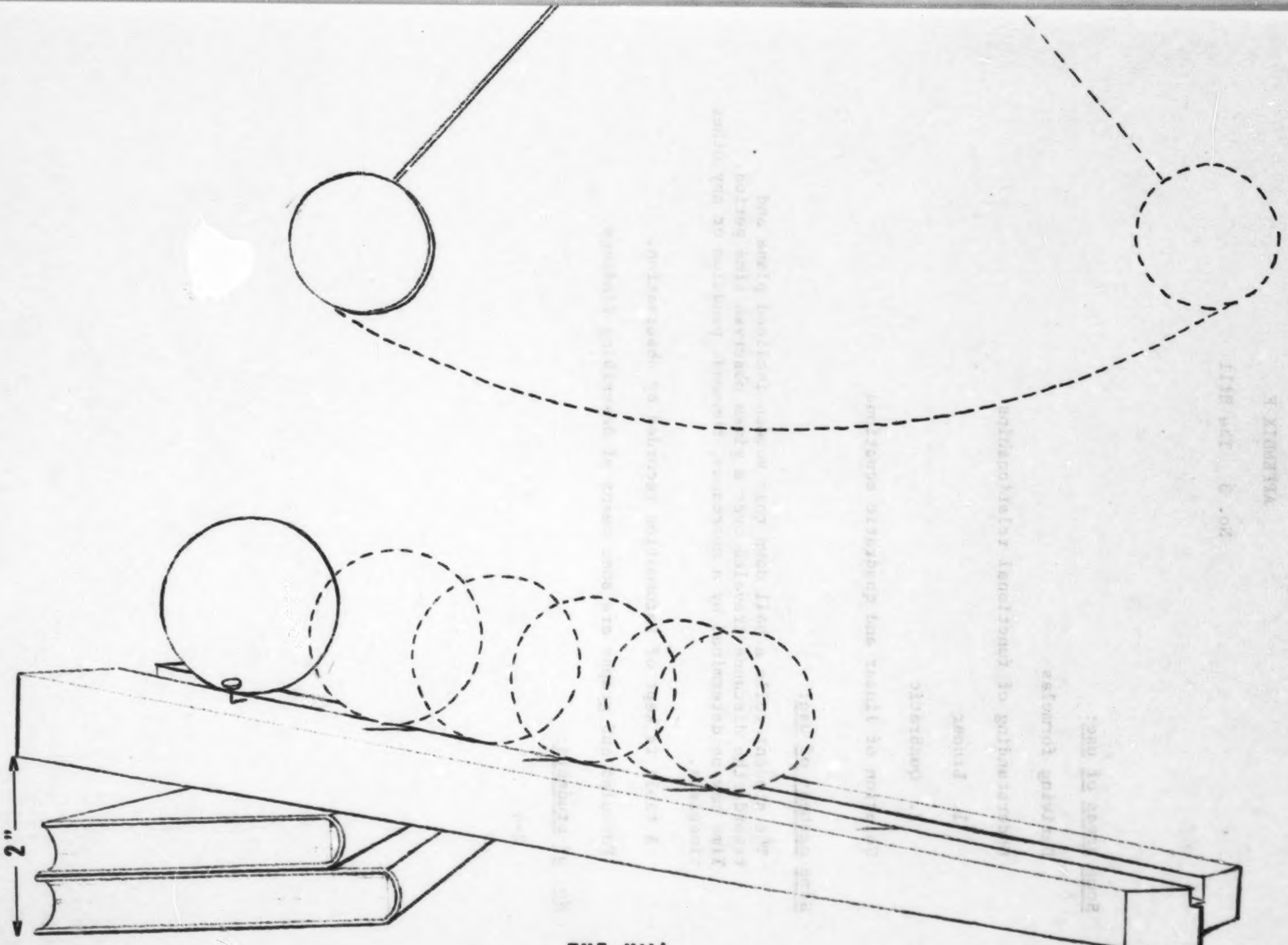
Some methods of use:

The board is made of wood with toy plastic gears completing the device. Each gear serves as an operator; i.e., multiplication by -1.

The student moves one slide a given distance resulting in a distance moved by the second slide. With one gear in use, the effect of multiplication by -1 is achieved. Two gears in use yields the effect of multiplication by +1 or (-1)(-1).

No. of students:

2 or 3



THE HILL

APPENDIX F

No. 3 The Hill

Some areas of use:

Deriving formulas

Understanding of functional relationships

1. Linear
2. Quadratic

Solution of linear and quadratic equations

Some methods of use:

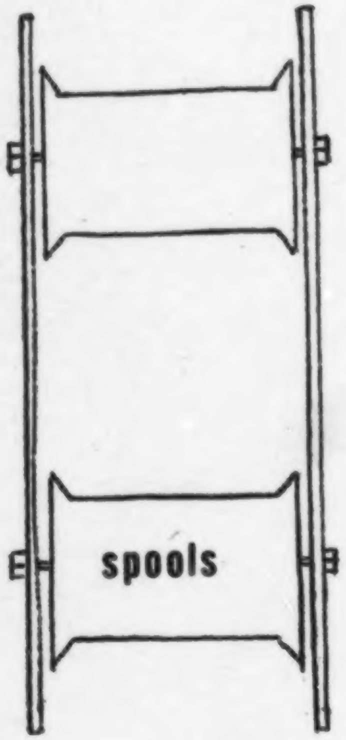
The student rolls a ball down this wooden inclined plane and records the distance traveled over a given observed time period. Time may be determined by a metronome, homemade pendulum or any other timepiece.

A table is kept of information recorded by observation.

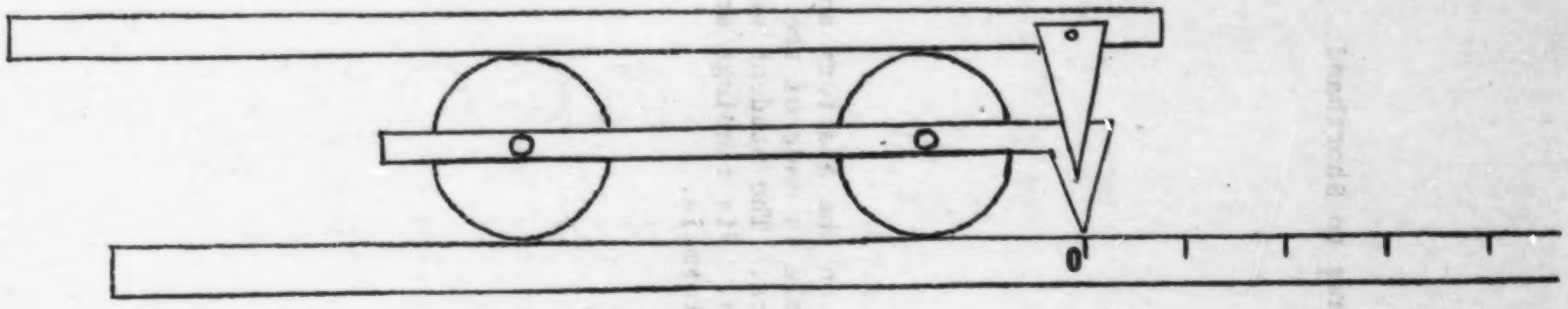
Formulas and graphs are some means of describing findings.

No. of students:

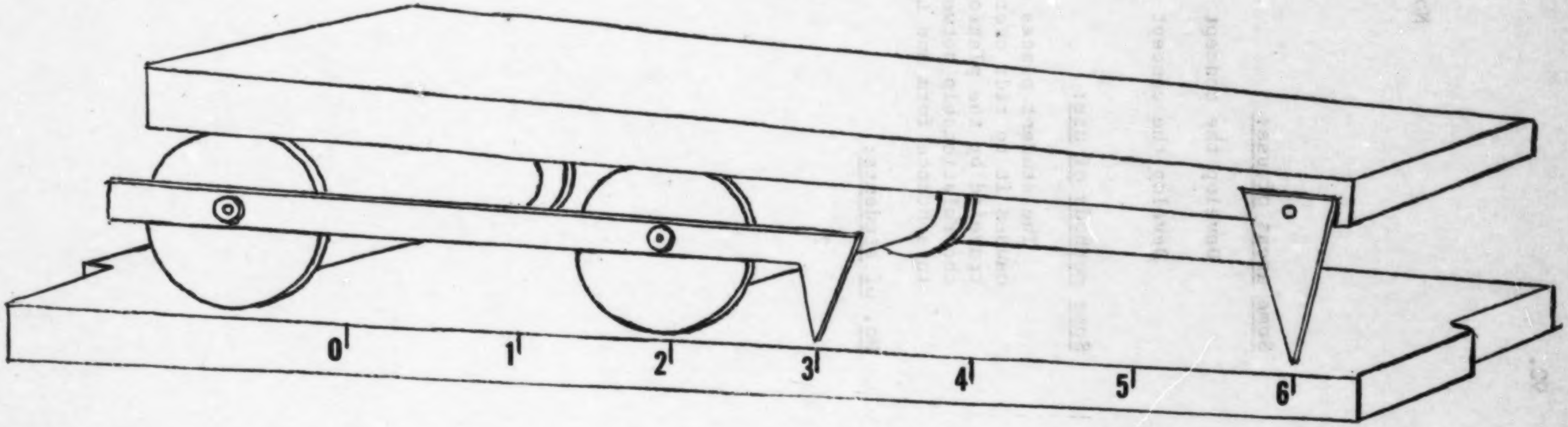
2-4



starting position



platform & rollers



APPENDIX F

No. 4 From Natural Laws to Shorthand

Some areas of use:

Develop the concept of formula

Develop the concept of linear function

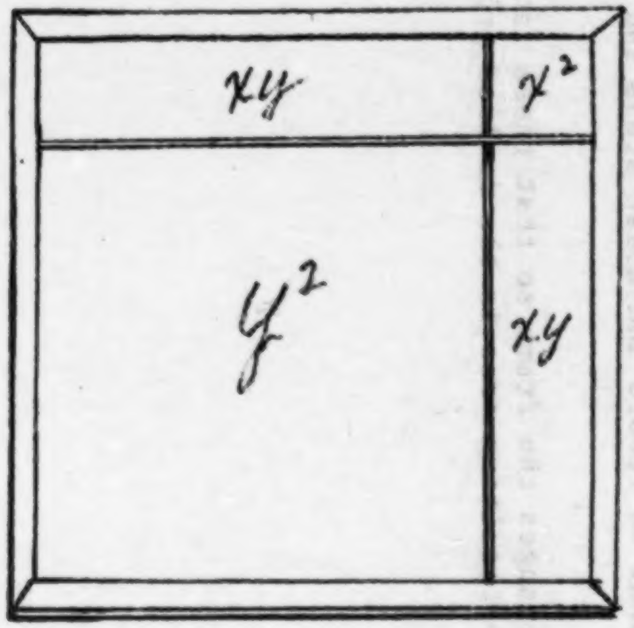
Some methods of use:

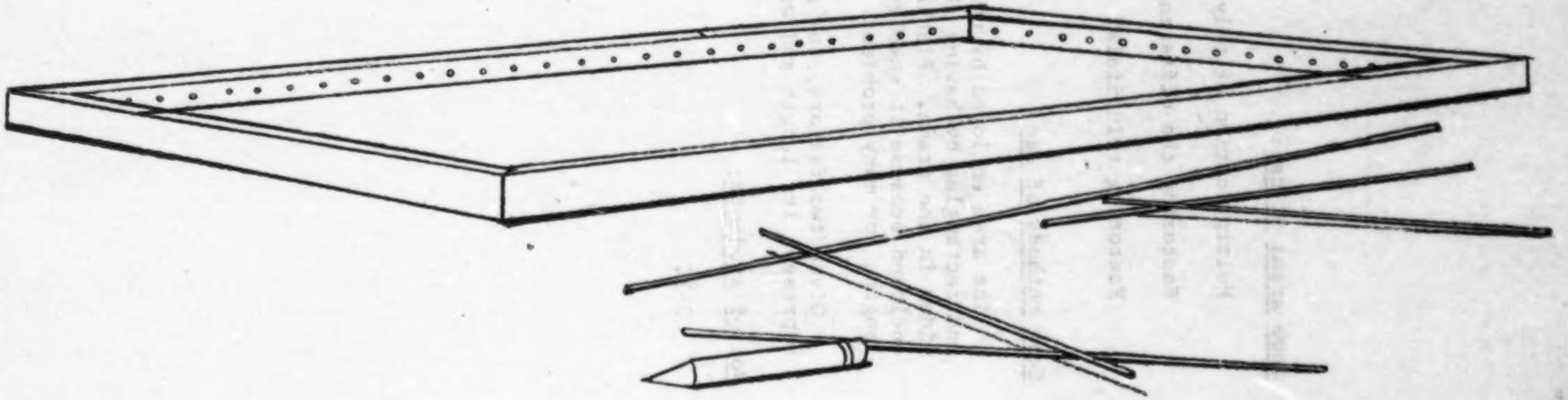
The student places his hand, palm down on the platform and causes it to ride over the rollers. A table is made of the distance traveled by the platform and by the rollers. The student seeks out the relationship between the two distances. His findings are written in sentence form and then shortened to a formula.

No. of students:

2

SOLVE: (X+Y)(X+Y)





FRAME

APPENDIX F

No. 5 The Frame

Some areas of use:

Multiplication of polynomials

Factoring the difference between 2 squares

Factoring trinomials.

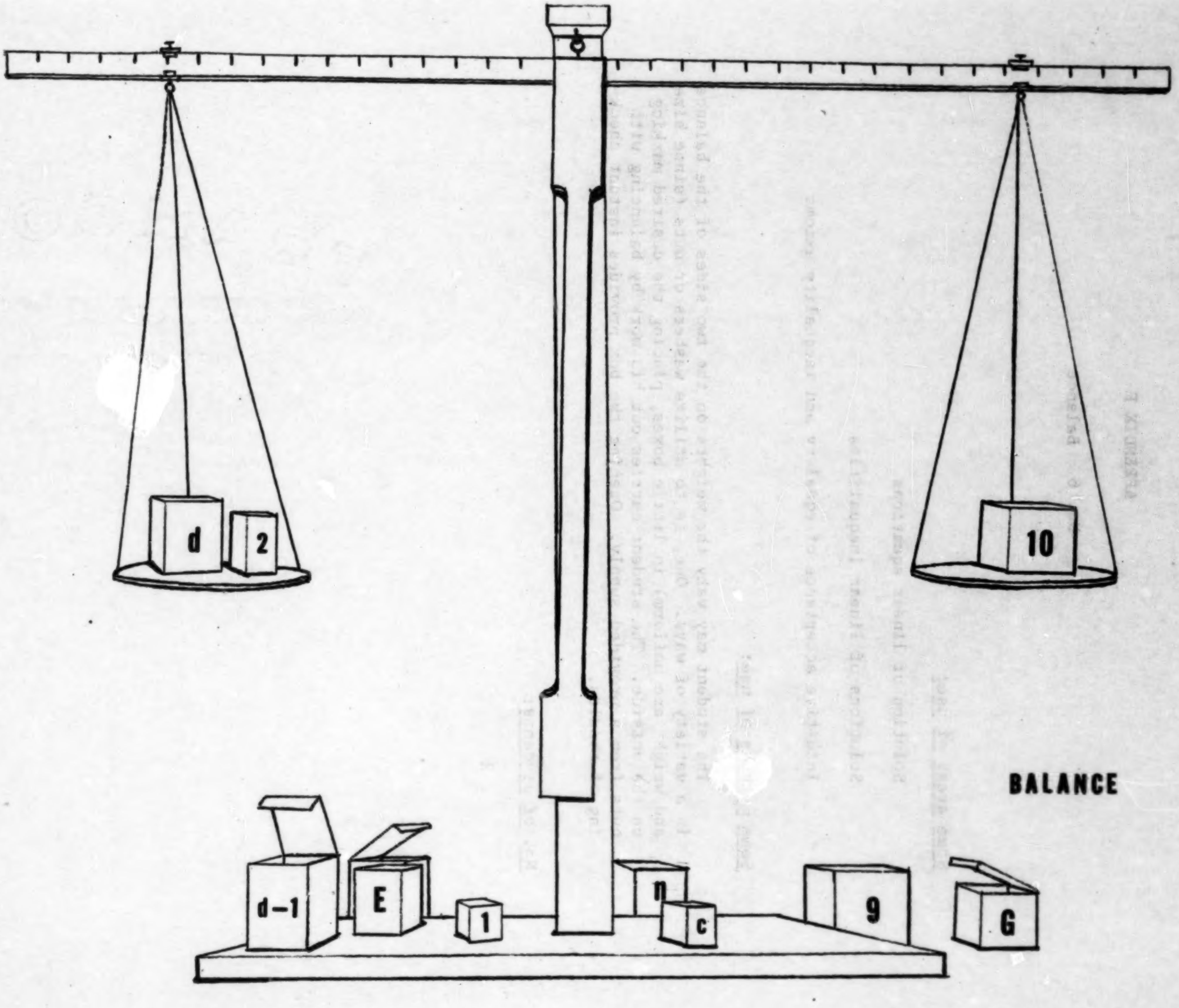
Some methods of use:

The area enclosed by the wooden frame is subdivided into squares and rectangles by having the student push rods through the holes provided in the frame. Blackboard surface is most convenient for the area enclosed because it enables the student to record and erase areas and lengths for many problems.

Given two factors, the student arranges the frame so that these factors represent its length and width. The resulting area is the desired product.

No. of students:

2-3.


d
2
10
d−1
E
1
n
c
9
G
BALANCE

APPENDIX F

No. 6 Balance

Some areas of use:

Solution of linear equations

Solutions of linear inequalities

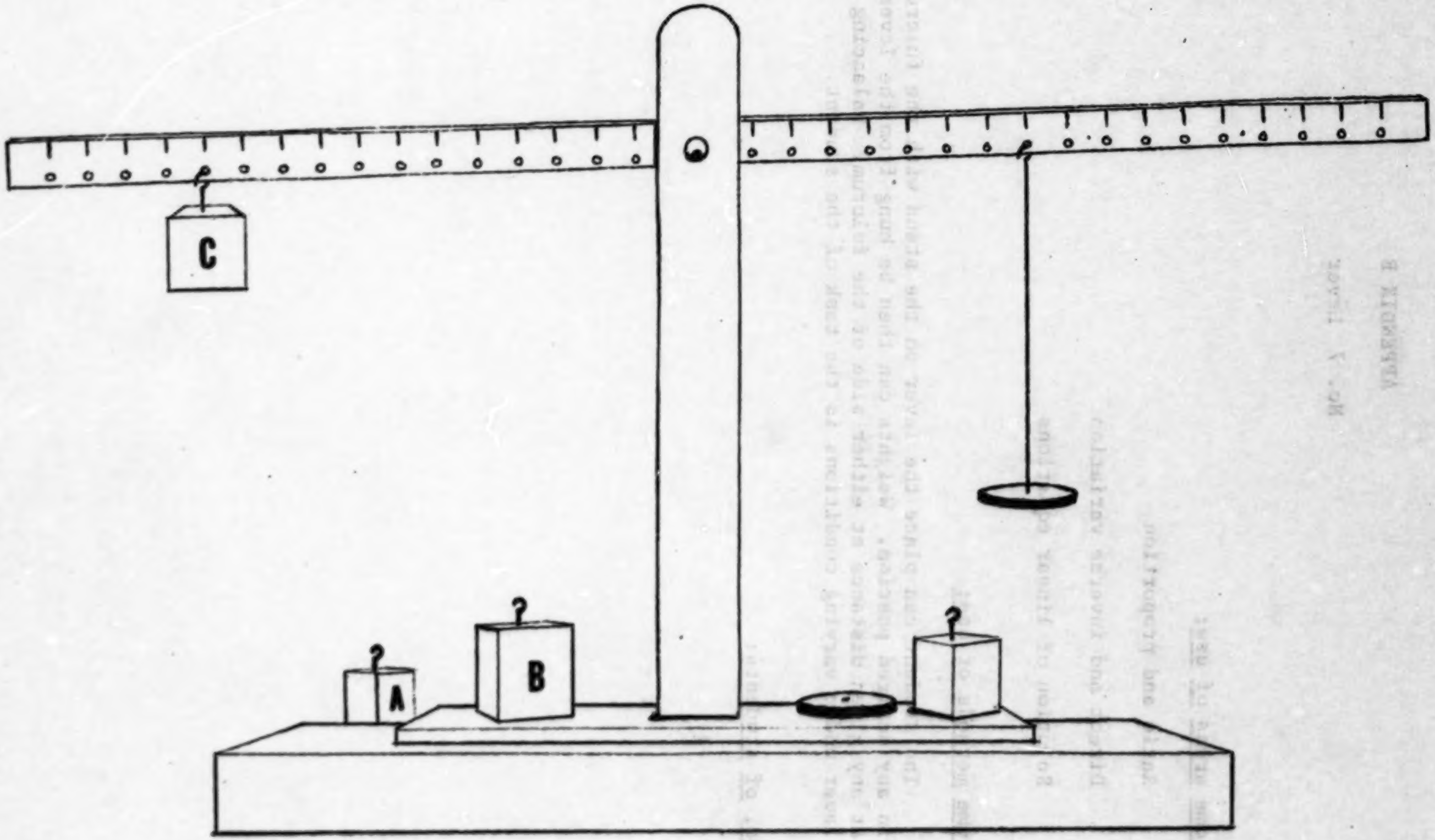
Intuitive acceptance of equality and inequality axioms

Some methods of use:

The student may vary the weights on the two sides of the balance in a variety of ways. One, is to utilize washers or nuts (since size and weight are uniform) in little boxes, placing the desired marking on the outside. The student carries out his work by balancing with nuts from a provided supply. Opening the box provides instant checking of results.

No. of students:

2



LEVER

APPENDIX F

No. 7 Lever

Some areas of use:

Ratio and proportion

Direct and inverse variation

Solution of linear equations

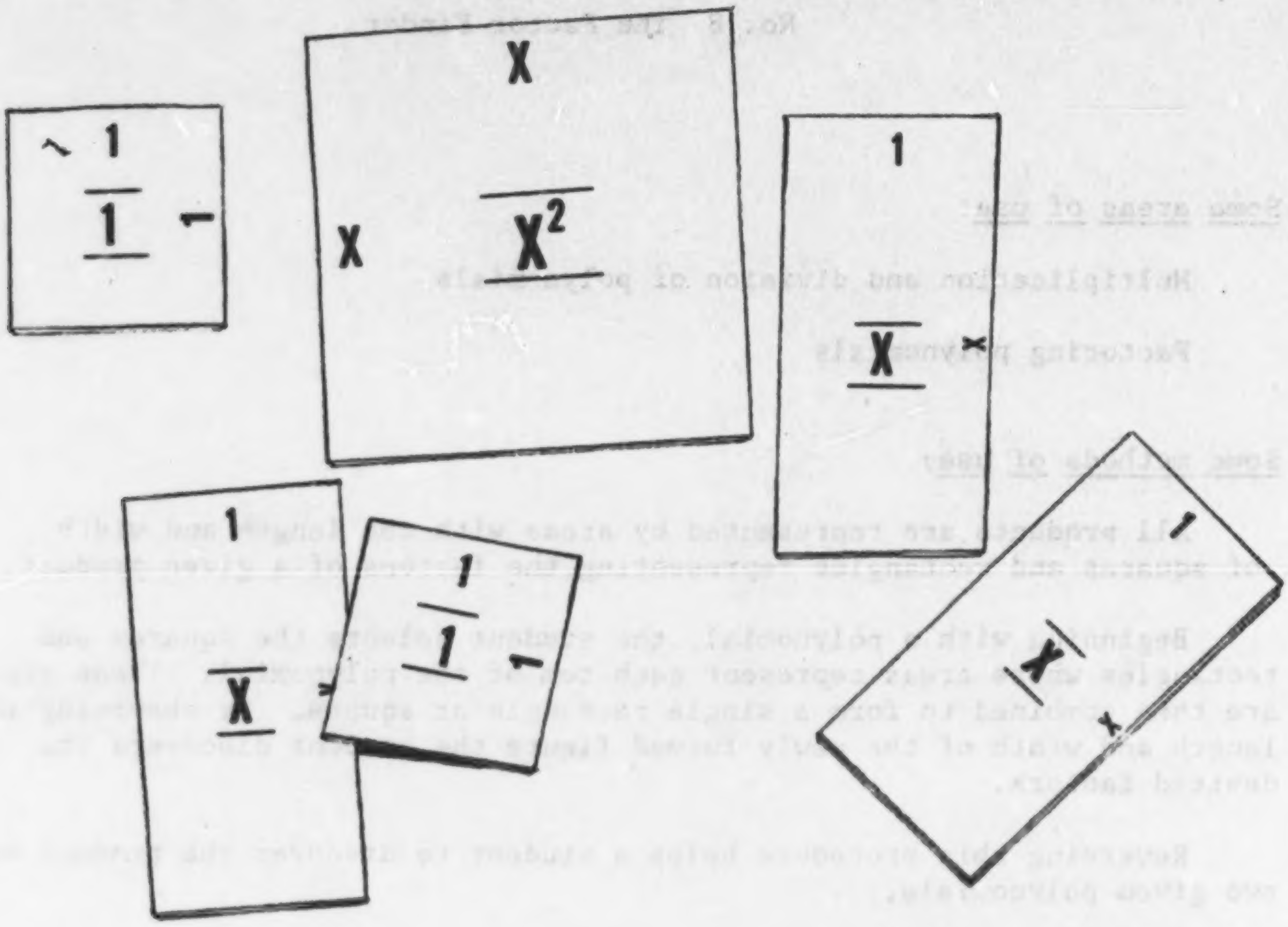
Some methods of use:

The student can place the lever on the stand with the fulcrum in any desired postion. Weights can then be hung from the lever at any given distance at either side of the fulcrum. Balancing the lever under varying conditions is the task of the student.

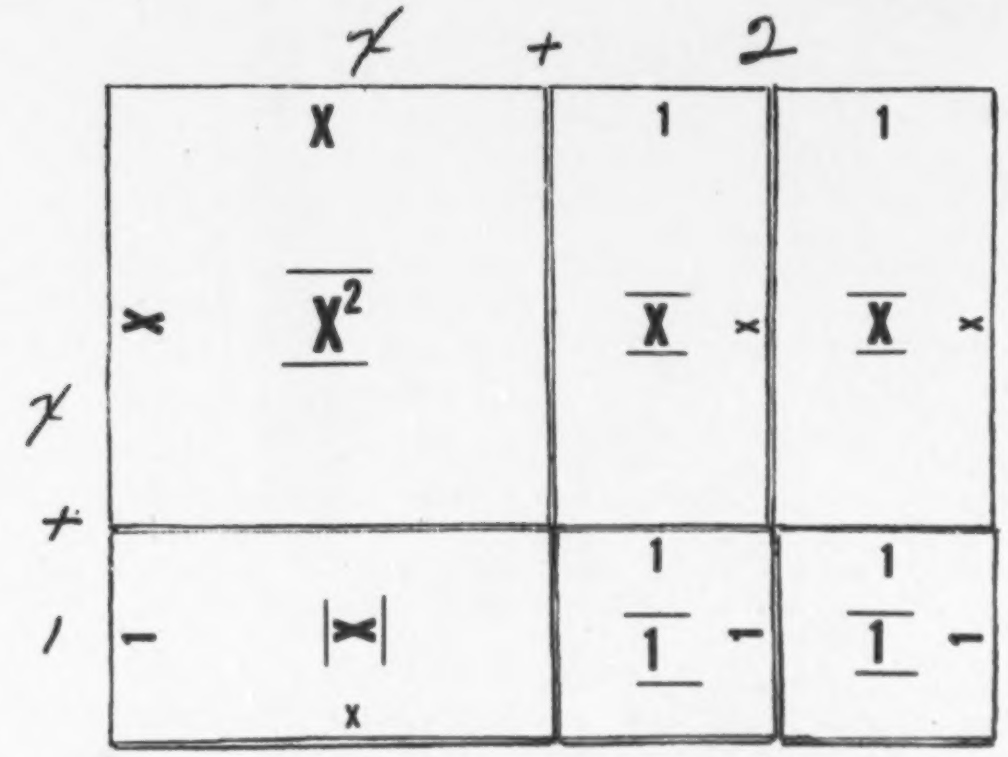
No. of students:

2

FACTOR: $X^2 + 3X + 2$


1
1
X
X
X²
1
X
1
X
1
1
1
X

FACTOR FINDER


x + 2
x + 1
X
X²
1
X
1
X
x
1
1

$$x^2 + 3x + 2 = (x+1)(x+2)$$

APPENDIX F

No. 8 The Factor Finder

Some areas of use:

Multiplication and division of polynomials

Factoring polynomials

Some methods of use:

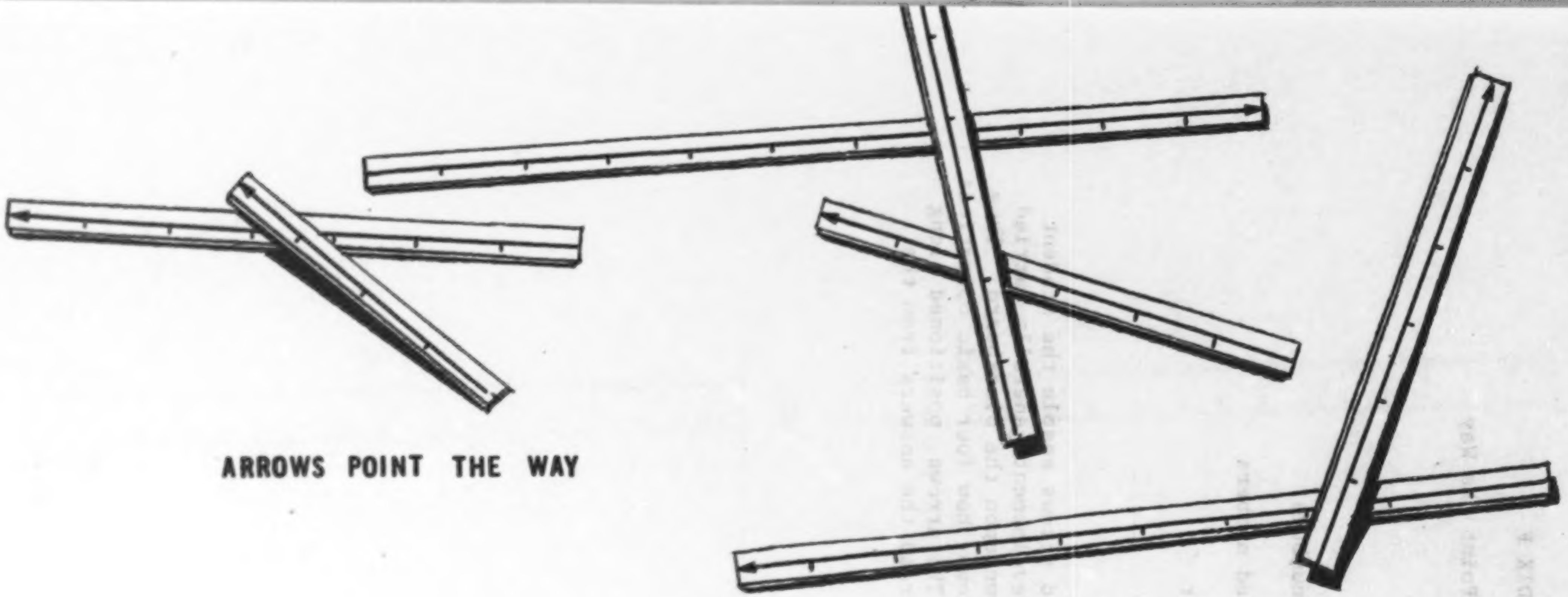
All products are represented by areas with the length and width of squares and rectangles representing the factors of a given product.

Beginning with a polynomial, the student selects the squares and rectangles whose areas represent each tem of the polynomial. These pieces are then combined to form a single rectangle or square. By observing the length and width of the newly formed figure the student discovers the desired factors.

Reversing this procedure helps a student to discover the product of two given polynomials.

No. of students:

2.

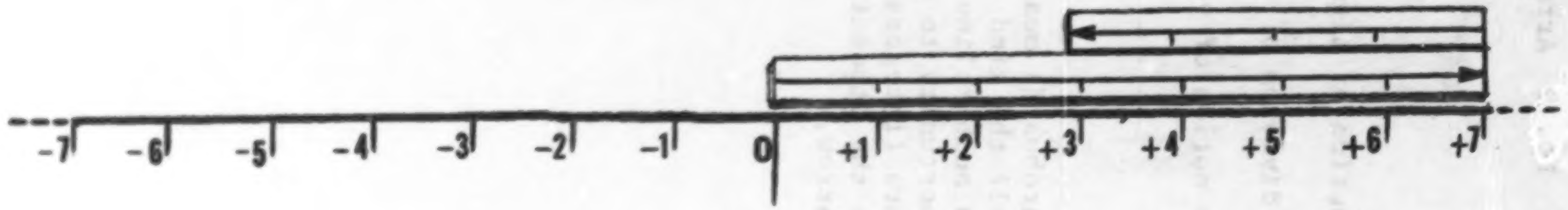


ARROWS POINT THE WAY

SOLVE

+7

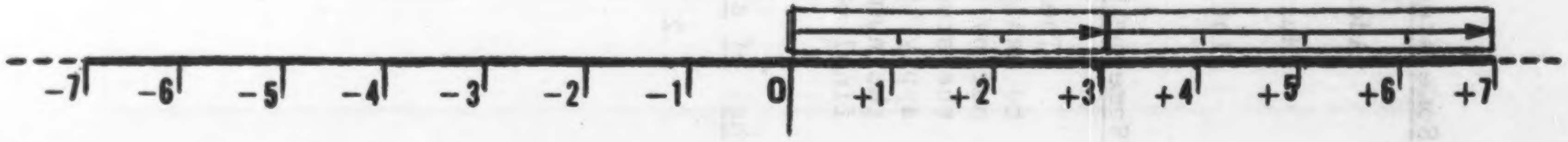
+ −4


−7
−6
−5
−4
−3
−2
−1
0
+1
+2
+3
+4
+5
+6
+7

SOLVE

+3

+ +4


−7
−6
−5
−4
−3
−2
−1
0
+1
+2
+3
+4
+5
+6
+7

APPENDIX F

No. 9 Arrows Point the Way

Some areas of use:

Addition and subtraction of signed numbers

Multiplication and division of signed numbers

Introduction to the notion of vector

Some methods of use:

These wooden (or cardboard) measured arrows enable the student to manipulate for himself the kind of development generally carried out by the teacher on a number line drawn upon the blackboard. Thus the student has the opportunity to discover how four basic operations apply to directed numbers (integers). The arrows, positioned along the number line, enable the student to read the answers from the final position of the arrow.

No. of students:

2

PRECEDING PAGES MISSING

MILE NEWSLETTER

Editor - Joan Abrams
Vol. I - No. 1
November 1960

Hello - This is the first official issue of our Newsletter, and I, a Junior at Queens College, am its first Editor. In introducting myself let me say that I am a prospective English teacher, who is very anxious to learn the techniques of education, and very eager to teach.

Unfortunately, I haven't yet met you, but I know all about you. I know you as pionners working under extremely difficult conditions - limited equipment, a shortage of manpower, and sometimes unfriendly neighbors. Although I cannot combat these factors, I will try to make life easier by setting up lines of communication amongst all of you. This is your Newsletter, and I am your Editor - and both of us are working for you. We shall try to keep you informed about meetings, special announcements, progress within the schools, and ideas you have sent in to be shared. Should you have any news you'd like published, please contact me any evening at Fieldstone 7-7813.

Your Editor,
Joan Abrams

WILL YOU HELP?

Remember when all this was started, and the only tools to be had were textbooks, and ingenuity, and team-*work*? Indeed, we have progressed since then. Where materials were lacking that could be bought commercially, e.g. tapes, cuisenaire rods, copies of tutor-texts, et al., many schools bought their own. In the interest of saving time, we suggest that you do the same. We will however, pay for any materials that your school is unable to buy. At the present time, we do not have any peg-boards. But we know you are ingenious enough to somehow improvise until ours come through.

MEMOS

Who has 'em. We need 'em. Please return to Herb Fremont or Gwladys Crosby:

Lockwood & Downs -
  Algebra
Durell, Palmer
  Elementary Algebra

****

Don't forget to send your class rosters labelled EXPERIMENTAL and CONTROLLED, to Harold Mitzel. Your tests are to have been sent express, collect, to him at 535 E. 80th Street, N.Y.C.

****

Remember Herb Fremont can be reached any morning before 11:00 A.M., at Walnut 1-5500. Having had three years' experience with Individualization, he has many ideas for all of you.

DR. SAWYER'S VISIT

On Oct. 19th Dr. Sawyer visited Alex Imre's School. He is expected to return to New York on Nov. 7th, at which time he will stop in at Brentwood.

If anyone wishes to have Dr. Sawyer visit him, please let us know, and we will try to arrange it.

****

The next meeting will be held on Monday, November 14th, in Remsen Lounge, Queens College.

MILE NEWSLETTER

LAST MONTH'S MEETING

At the meeting held on Oct. 10th, at Syosset High School, we tried to make arrangements for the exchange of tests. Art handed out memo'd copies of three tests he had used. Do you have any you could bring to the next meeting?

Later, tapes of some lessons were distributed. Others will be forthcoming as soon as they are completed. When the meeting was over, we all had the feeling that it would be helpful to get together and talk over our very numerous and assorted problems. For example, we learned that if you make a pendulum 39" long it will swing once per second, and thus enable students to use The Hill without a metronome. Any other suggestions?

In addition to our meetings, we hope that this Newsletter will provide another means for exchanging ideas.

Lost, Misplaced, or in Process

#3. Gear Board (Any idea when we will have it?)
#4. Pegboard (Any success?)
#6. Has anyone part I?
#8. Seeing Numbers
#28. Making Formulas (we need this one badly; if anyone has any idea of its whereabouts please phone us.)
#35. Balance (written material)
Exp. #5 (Revised copy)
Exp. #6. Thermo-Meters and Bargraph.

RE:UNITS

Listed below are the units that have already been sent out. If there are any units that you haven't yet received, please contact Rose Landa at Queens College, HI 5-7500, Ext. 312 and she will see that you get them.

1. The Hill
1a. The Hill
2. An experiment with cardboard
3. Gear Board
3a. Gear Board
3b. Gear Board
4. Numbers you can see
5. Keeping Equals
6. Intro. to Cuisenaire Rods.
7a. Ditto
7b. Ditto
9. The Pythagorean Theorem
10. Trig.
11. Difference of Two Squares
15. The shape of numbers
16. Operations with signed numbers
17. Scrabble Solved Systems
18. Cross number puzzle
19. Picture Puzzle
20. Multiplying with Rods
21. Cuisenaire Rods for Linear Equations
22. Solving Systems with Rods
23. Cuisenaire Rods for Factoring
25. Proportions of Lines
26. Indirect Measurement
27. Cuisenaire Rods in the Fundamental Operations
30. Changing right triangles
31. Trig II
32. Cuisenaire Rods to Understand Ratio
33. Fractions
39. Introductions to Trig Advanced
40. Water Spout
45. Solving Equations with Scrabbl
49. Un-things

You Should also have:

1. The TMI Program
2. The Temac Program(Brit.)
3. Three tapes
4. Ten copies of Sheila's cards in motion
5. Ten of Sheila's Cards for practice ONLY
6. Hill
7. Double Wheel
8. Pulley
9. Balance
10. Lever
11. Swinging Rod
12. Five S.M.S.G. Programs
13. Math. Patterns in Science
14. Herb's Math in Motion
15-30 Power-finder

MILE NEWSLETTER

WE ARE NOT ALONE

Below is an article as it appeared in the New York Times, on Wednesday, October 26th. Somehow, it makes one feel good to know that the people who have been turning out fain carbon copies of students are beginning to realize that there is something quite wrong with their machine. At the present time, we can only work and wait and hope. And there is hope indeed because our team is daily growing larger.

TEACHERS WARNED OF INFLEXIBILITY

> Syracuse, Oct. 25th - The nation's teachers were chided today for generally failing to adapt their teaching practices to the fact that children of the same age are different in many ways. Dr. William D. Firman, an official of the State Education Department, said that as a result most children "are still organized by chronological age, seated in neat rows, given identical textbooks, taught primarily by the lecture method, given the same test at the end of the year, and uniformly promoted."
>
> "These practices,"he said,"fail to recognize that there are great variations in childrens' abilities, interests, attitudes, motivations, and in how and what they learn.
>
> Dr. Firman spoke at a meeting of members of boards of education from cities throughout the state, which followed the close of the annual convention here at the State School Boards Association. The session was held at the Hotel Onondaga. Dr. Firman said that quality in education required schools to provide for the individual differences of their pupils, as well as to meet other criteria.

* *

IN THE MEANTIME, if you have any brainstorms, any suggestions, comments or circiticisms, any problems we can solve, any questions we can answer - and most of all any ideas you would like to share with your co-workers, please call me (Fieldstone 7-7813), and I will see that it is properly taken care of. During the day, call Mrs. Landa at HI 5-7500, Ext. 312.

* *

Until then, I am looking forward to seeing all of you at the next meeting on Monday, November 14th at 4:30 P.M. ,in Remsen Lounge, Queens College.